A PRACTICAL TABULAR METHOD FOR CALCULATING FUSELAGE FRAMES OF THE SEMI-MONOCOQUE TYPE LOADED BY r ARBITRARY VERTICAL FORCES

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Let a fuselage frame, of an arbitrary shape and with a vertical axis of symmetry, exist at the station of the fuselage shown in figure 1. The frame is loaded by vertical forces F_1, F_2, ... F_r arbitrarily distributed on its periphery (figure 2). It is necessary to check the dimension accuracy of the frame.

The frame is three times statically indeterminate because we consider the problem in a plane, and by cutting the frame for obtaining a statically determinate system three unknown quantities appear: N_A, T_A and M_A (axial load, shear load and bending moment). By partially differentiating the strain energy of the frame

$$2\,W = \oint \frac{N^2}{EA}\,.\,dl + \oint \frac{T^2}{GA}\,.\,dl + \oint \frac{M^2}{EJ}\,.\,dl$$

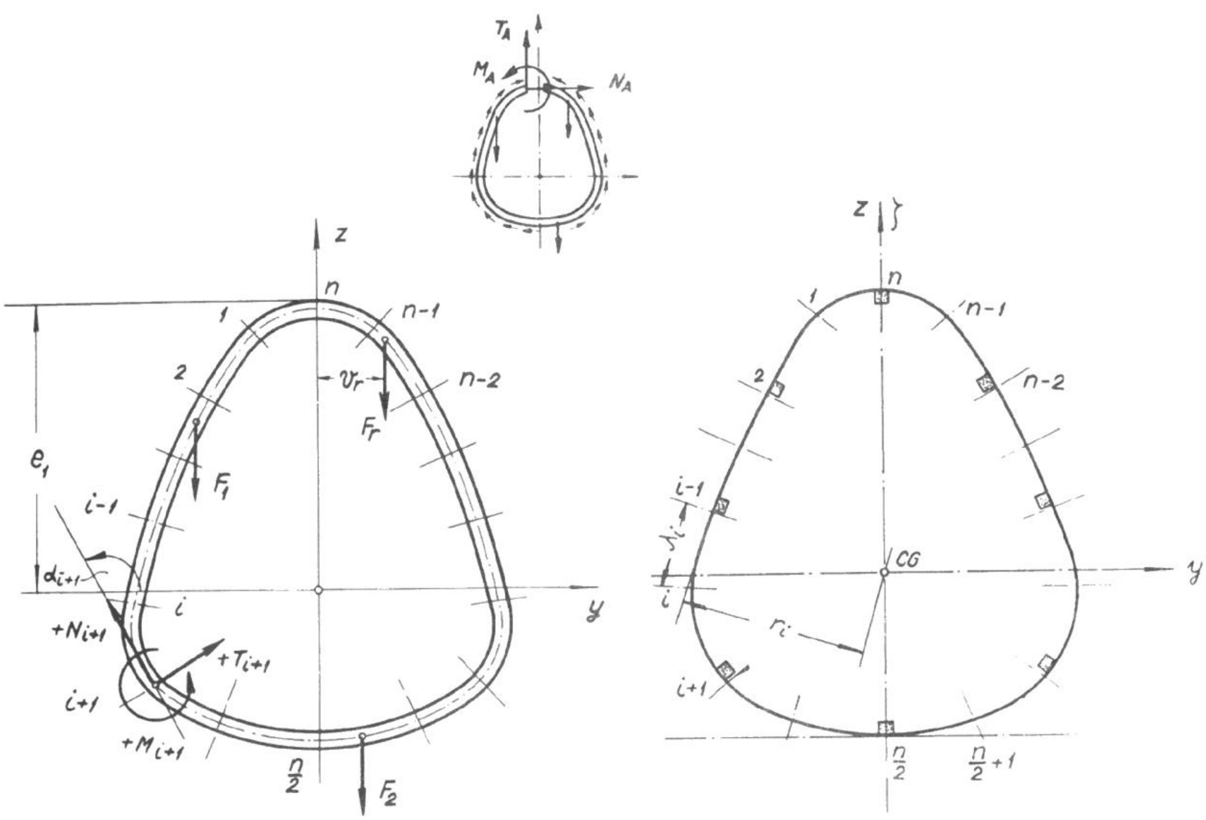

Figure 1

with respect to the unknown quantities and setting these derivatives equal to zero we obtain three equations for calculating three unknown quantities:

$$\oint \frac{\partial N}{\partial N_A} \cdot \frac{N\,dl}{E\,.\,A} + \oint \frac{\partial T}{\partial N_A} \frac{T.\,dl}{GA} + \oint \frac{\partial M}{\partial N_A} \cdot \frac{M\,dl}{EJ} = 0 \quad \ldots\ldots (1)$$

$$\oint \frac{\partial N}{\partial T_A} \cdot \frac{N.\,dl}{EA} + \oint \frac{\partial T}{\partial T_A} \cdot \frac{T\,dl}{GA} + \oint \frac{\partial M}{\partial T_A} \cdot \frac{M\,dl}{EJ} = 0 \quad \ldots\ldots (2)$$

$$\oint \frac{\partial M}{\partial M_A} \cdot \frac{M\,dl}{EJ} = 0 \quad \ldots\ldots (3)$$

In the above equations

$$\left.\begin{aligned} N &= N_o + N_a \cdot \cos\alpha + T_A \cdot \sin\alpha \\ T &= T_o + N_a \cdot \sin\alpha - T_A \cos\alpha \\ \text{and } M &= M_o - T_A \cdot y - N_A(e_1 - z) + M_A \end{aligned}\right\} \quad \ldots\ldots (4)$$

E represents the modulus of elasticity of the material of which the frame is made, G the shearing modulus of elasticity, A the area and J the moment of inertia of the cross section of the frame about an axis parallel to x and passing through its center of gravity. In fact the area and the moment of inertia of the cross section of the frame should include a certain effective width of the skin on either side of the frame as well. However in absence of the exact data as to the quantity of this width one can consider the area and the moment of inertia of the cross section of the frame considered only. y and z are the coordinates of the points of the corresponding sections on the gravity line of the frame and α the angle between the positive y-axis and the tangent drawn on to gravity line, oriented always from the ν to the $(\nu - 1)$ section.

To find the values N_o, T_o and M_o, besides the known external forces F_1, F_2, ... F_r it is necessary to know the shear flow t which loads the frame and which is equal to the sum of the shear flows t^T and t^M. These shear flows result from the difference of shear forces of the fuselage directly before and behind the frame,

$$\Delta T = \sum_{\nu=1}^{r} F_\nu$$

and from the difference of the corresponding torsional moments

$$\Delta M_t = \sum_{\nu=1}^{r} F_\nu \cdot v_\nu$$

respectively.

We find the shear flow t by the means of the table T 1, (see page 124) where:

- δ = thickness of the skin of the fuselage;
- A_{ik} = area of the skin in the i-th part of the cross section of the fuselage;
- A_{is} = area of the cross sections of the stringers in the i-th part of the cross section of the fuselage;
- ζ_{ik} = distance from the center of gravity of the skin in the i-th part to the η-axis;
- ζ_{is} = distance from the center of gravity of the stringers in the i-th part to the η-axis.

Significances of other symbols can be seen from figure 1. λ_i is quite arbitrary and can be variable along the outline.

The centroid of the cross section of the fuselage ζ_o can be found by means of the columns 2, 3, 6 and 7 by the expression

$$\zeta_o = \frac{\sum_{i=1}^{n/2} A_{ik} \cdot \zeta_{ik} + \sum_{i=1}^{n/2} A_{is} \cdot \zeta_{is}}{\sum_{i=1}^{n/2} A_{ik} + \sum_{i=1}^{n/2} A_{is}} = \frac{\Sigma 6 + \Sigma 7}{\Sigma 2 + \Sigma 3}$$

and the moment of inertia by the means of the columns 10 and 11, i.e.

$$I_y = 2\left[\sum_{i=1}^{n/2} A_{ik} \cdot z_{ik}^2 + \sum_{i=1}^{n/2} A_{is} \cdot z_{is}^2\right] = 2(\Sigma\ 10 + \Sigma\ 11).$$

For obtaining a statically determinate system we assume that the outline is cut at station n, thus the static moments of different parts of the cross section of the fuselage considered are:

$$(S_y)_i = \sum_{\nu=1}^{\nu=i} A_{\nu k} \cdot z_{\nu k} + \sum_{\nu=1}^{i} A_{\nu s} \cdot z_{\nu s} = (S_y)_{i-1} + A_{ik} \cdot z_{ik} + A_{is} \cdot z_{is} \qquad [\text{column } 14]$$

Therefore the shear flow t_i^T is

$$t_i^T = \frac{\Delta T}{I_y} \cdot (S_y)_i = \frac{\Delta T}{\ldots} \cdot 14$$

and is given in the column 15.

When determining the shear flow t_i^T it is assumed that there is no buckling of the skin of the fuselage, and therefore in the expressions for J_y and S_y the whole skin is taken fully effective. In the case of appearing of the buckling of the stressed-skin for the given load, which is frequently the case in practice, instead of the whole widths of the stretches between stringers, the effective widths are to be taken. They are obtained after a few successive approximations applied in determining the normal stresses along the contour of the considered cross section of the fuselage. Since the error of the values of the shear flows, in case the effective widths are not taken into account is practically small, we can assume, with enough accuracy, that no buckling of the skin adjacent to the frame occurs in frame calculating.

We take the positive sign of the force F_y if the force is directed upward and the negative sign if it is directed downward. Since in this problem we consider the resisting shear flows, the obtained negative flows correspond to the clockwise sense, the positive shear flows to the counter-clockwise sense.

The flow resulting from the difference of the torsional moments and passing on the frame is

$$t_i^M = \frac{\Delta M_t}{2A}$$

and is given in column 18. Here A represents the area bounded by the outline of the cross section of the fuselage considered:

$$A = \sum_{i=1}^{n/2} \lambda_i \cdot r_i = \Sigma\ 17 .$$

We take the clockwise moment ΔM_t for positive, i.e. the obtained positive torsional resisting shear flow correspond to the counterclockwise sense.

The resulting flow

$$t = t^T + t^M$$

is given in column 19.

In the following consideration we assume that the elementary resulting forces ΔF_i of the shear flow t of any part "i" act upon the section "i", and that they are equal

$$\Delta F_i = t_i^m \cdot \lambda_i ,$$

where

$$t_i^m = \frac{t_{i-1} + t_i}{2}$$

It is obvious that the error is smaller if the lengths are smaller, i.e. if their number along the counter of the cross section becomes greater.

Horizontal and vertical components of these forces ΔF_{yi} and ΔF_{zi} are given in columns

25 and 26 in the table T2, and are obtained by the expressions (see page 125)

$$\Delta F_{yi} = - \Delta F_i \, . \, \cos \alpha_i \quad \text{and}$$

$$\Delta F_{zi} = - \Delta F_i \, . \, \sin \alpha_i$$

Negative signs in the last two expressions are introduced in order to bring in accordance the signs of forces with chosen axis orientations (y, z) and with chosen positive and negative senses of shear flows as well.

Using the columns 27 to 35 we obtain in the column 36 the value of the axial force of the statically determinate system

$$N_{oi} = \cos \alpha_i \sum_{\nu=1}^{\nu=i} \Delta F_{y\nu} + \sin \alpha_i \sum_{\nu=1}^{\nu=i} \Delta F_{z\nu} + \sin \alpha_i \sum_{k=1}^{q} F_k$$

$$= a_i \cos \alpha_i + b_i \sin \alpha_i + \sin \alpha_i \sum_{k=1}^{q} F_k,$$

and in the column 37 the value of the corresponding shear force too

$$T_{oi} = \sin \alpha_i \sum_{\nu=1}^{\nu=i} \Delta F_{y\nu} - \cos \alpha_i \sum_{\nu=1}^{\nu=i} \Delta F_{z\nu} - \cos \alpha_i \sum_{k=1}^{q} F_k$$

$$= a_i \sin \alpha_i - b_i \cos \alpha_i - \cos \alpha_i \sum_{k=1}^{q} F_k$$

The moment M_o for section *i* is given in the column 45, i.e.

$$M_{oi} = M_{o(i-1)} - (z_{i-1} - z_i) \sum_{\nu=1}^{\nu=i-1} \Delta F_{y\nu} - (y_i - y_{i-1}) \, . \sum_{\nu=1}^{\nu=i-1} \Delta F_{z\nu} - \sum_{k=p}^{q} F_k (y_i - y_k) - (y_i - y_{i-1}) \, . \sum_{k=1}^{p-1} F$$

$$= M_{o(i-1)} - (z_{i-1} - z_i) \, . \, a_{i-1} - (y_i - y_{i-1}) \, . \, b_{i-1} - \sum_{k=p}^{q} F_k (y_i - y_k) - 41 \, . \sum_{k=1}^{p-1} F_k$$

(In the case that besides the external vertical forces the frame is also loaded by external bending moments, it is necessary to add to the expression for M_{oi} the sum of all external moments from the section *n* to the section *i*, taken in a counter-clockwise sense).

The sum $\sum_{k=1}^{q} F_k$ in the above equations is taken with respect to all external forces F_r. This sum involves for the section *i* q external forces, which act between the sections *n* and *i*, taken in a counter-clockwise sense. In the case that there are no forces for the section *i*, the sum $\sum_{k=1}^{q} F_k$ is equal to zero. For the section *i* the sum $\sum_{k=p}^{q} F_k \, . \, (y_i - y_k)$ involves (q - p) external forces, which act between the sections (i - 1) and i. If there are no external forces in the section *i*, $\sum_{k=p}^{q} F_k \, . \, (y_i - y_k)$ for the section *i* is equal to zero.

Since

$$\frac{\partial N_i}{\partial N_A} = \cos \alpha_i, \quad \frac{\partial T_i}{\partial N_A} = \sin \alpha_i, \quad \frac{\partial M_i}{\partial N_A} = - (e_1 - z_i),$$

$$\frac{\partial N_i}{\partial T_A} = \sin \alpha_i, \quad \frac{\partial T_i}{\partial T_A} = - \cos \alpha_i, \quad \frac{\partial M_i}{\partial T_A} = - Y_i \quad \text{and} \quad \frac{\partial M_i}{\partial M_A} = 1,$$

it follows

$$\oint \frac{\partial N}{\partial N_A} \, . \, \frac{N \, dl}{EA} = \frac{1}{EA} \left\{ \sum_{i=1}^{n} \lambda_i N_{oi} \cos \alpha_i + N_A \sum_{i=1}^{n} \lambda_i \cos^2 \alpha_i + T_A \sum_{i=1}^{n} \lambda_i \cos \alpha_i \sin \alpha_i \right\}$$

$$= \frac{1}{EA}\left\{ \Sigma\, 46 + N_A\, \Sigma\, 47 + T_A \cdot \Sigma\, 48 \right\}.$$

$$\oint \frac{\partial T}{\partial N_A} \cdot \frac{T\,dl}{GA} = \frac{1}{GA}\left\{ \sum_{i=1}^{n} \lambda_i\, T_{oi} \sin\alpha_i + N_A \sum_{i=1}^{n} \lambda_i \sin^2\alpha_i - T_A \sum_{i=1}^{n} \lambda_i \cos\alpha_i \sin\alpha_i \right\}$$

$$= \frac{1}{GA}\left\{ \Sigma\, 49 + N_A\, \Sigma\, 50 - T_A\, \Sigma\, 48 \right\},$$

$$\oint \frac{\partial M}{\partial N_A} \cdot \frac{M\,dl}{EJ} = \frac{1}{EJ}\left\{ -\sum_{i=1}^{n} \lambda_i\, M_{oi}\, (e_1 - z_i) + N_A \sum_{i=1}^{n} (e_1 - z_i)^2 \cdot \lambda_i + \right.$$

$$\left. + T_A \sum_{i=1}^{n} (e_1 - z_i)\, y_i\, \lambda_i - M_A \sum_{i=1}^{n} (e_1 - z_i)\, \lambda_i \right\} =$$

$$= \frac{1}{EJ}\left\{ -\Sigma\, 53 + N_A\, \Sigma\, 54 + T_A\, \Sigma\, 61 - M_A\, \Sigma\, 52 \right\},$$

$$\oint \frac{\partial N}{\partial T_A} \cdot \frac{N dl}{EA} = \frac{1}{EA}\left\{ \sum_{i=1}^{n} \lambda_i\, N_{oi} \sin\alpha_i + N_A \sum_{i=1}^{n} \lambda_i \cos\alpha_i \sin\alpha_i + T_A \sum_{i=1}^{n} \lambda_i \sin^2\alpha_i \right\}$$

$$= \frac{1}{EA}\left\{ \Sigma\, 55 + N_A\, \Sigma\, 48 + T_A\, \Sigma\, 50 \right\}.$$

$$\oint \frac{\partial t}{\partial T_A} \cdot \frac{T\,dl}{GA} = \frac{1}{GA}\left\{ -\sum_{i=1}^{n} \lambda_i\, T_{oi} \cos\alpha_i - N_A \sum_{i=1}^{n} \lambda_i \sin\alpha_i \cos\alpha_i + T_A \sum_{i=1}^{n} \lambda_i \cos^2\alpha_i \right\}$$

$$= \frac{1}{GA}\left\{ -\Sigma\, 56 - N_A\, \Sigma\, 48 + T_A\, \Sigma\, 47 \right\},$$

$$\oint \frac{\partial M}{\partial T_A} \cdot \frac{M\,dl}{EJ} = \frac{1}{EJ}\left\{ -\sum_{i=1}^{n} \lambda_i\, y_i\, M_{oi} + T_A \sum_{i=1}^{n} \lambda_i\, y_i^2 + N_A \sum_{i=1}^{n} \lambda_i\, y_i\, (e_1 - z_i) - M_A \sum_{i=1}^{n} \lambda_i\, y_i \right\} =$$

$$= \frac{1}{EJ}\left\{ -\Sigma\, 58 + T_A\, \Sigma\, 59 + N_A\, \Sigma\, 61 - M_A\, \Sigma\, 57 \right\}$$

and

$$\oint \frac{\partial M}{\partial M_A} \cdot \frac{M\,dl}{EJ} = \frac{1}{EJ}\left\{ \sum_{i=1}^{n} M_{oi}\, \lambda_i - T_A \sum_{i=1}^{n} y_i\, \lambda_i - N_A \sum_{i=1}^{n} \lambda_i\, (e_1 - z_i) + M_A \sum_{i=1}^{n} \lambda_i \right\} =$$

$$= \frac{1}{EJ}\left\{ \Sigma\, 60 - T_A\, \Sigma\, 57 - N_A\, \Sigma\, 52 + M_A\, \Sigma\, 1 \right\}.$$

In the above expressions the values E, G, A and J are taken before brackets, assuming they are constant along the frame, which is usually the case encountered in practice. Otherwise, these values remain under the sums, being variable from section to section.

(For instance, if only the cross sections are variable along the frame, in the columns 46 to 50, 55 and 56 instead of λ_i we put $\frac{\lambda i}{A_i^m}$ and in the columns 52 to 54, 57 to 60 and also in the sum $\Sigma 1$ in the last equation instead of λ_i we put $\frac{\lambda i}{I_i^m}$, where

$$A_i^m = \frac{A_{i-1} + A_i}{2} \quad \text{and} \quad I_i^m = \frac{I_{i-1} + I_i}{2}$$

Therefore in the last expressions for linear integrals the quantities A and J being variables, are introduces under the sums, as shown above. Before the brackets remain only either modulus of elasticity or shearing modulus of elasticity).

Substituting linear integrals in the equations 1, 2 and 3 with obtained expressions, we get two equations with two unknowns N_A and M_A and one equation with the unknown T_A, for the sums $\Sigma\, 48$, $\Sigma\, 61$ and $\Sigma\, 57$ are equal to zero following the simmetry of the cross section, i.e.

$$N_A\left[\frac{\Sigma\,47}{EA}+\frac{\Sigma\,50}{GA}+\frac{\Sigma\,54}{EJ}\right]-M_A\frac{\Sigma\,52}{EJ}=\frac{\Sigma\,53}{EJ}-\frac{\Sigma\,46}{EA}-\frac{\Sigma\,49}{GA}, \qquad (5)$$

$$T_A\left[\frac{\Sigma\,50}{EA}+\frac{\Sigma\,47}{GA}+\frac{\Sigma\,59}{EJ}\right]=\frac{\Sigma\,56}{GA}+\frac{\Sigma\,58}{EJ}-\frac{\Sigma\,55}{EA} \qquad (6)$$

$$N_A\frac{\Sigma\,52}{EJ}-M_A\frac{\Sigma_1}{EJ}=\frac{\Sigma\,60}{EJ} \qquad (7)$$

Therefore from the equation 6 it follows

$$T_A=\frac{\dfrac{\Sigma\,56}{GA}+\dfrac{\Sigma\,58}{EJ}-\dfrac{\Sigma\,55}{EA}}{\dfrac{\Sigma\,50}{EA}+\dfrac{\Sigma\,47}{GA}+\dfrac{\Sigma\,59}{EJ}}$$

and from the equations 5 and 7

$$N_A=\frac{-C_2D_1+C_1D_2}{-B_1C_2+B_2C_1} \text{ and } M_A=\frac{B_1D_2-B_2D_1}{-B_1C_2+B_2C_1}$$

if we put

$$B_1=\frac{\Sigma\,47}{EA}+\frac{\Sigma\,50}{GA}+\frac{\Sigma\,54}{EJ},\quad B_2=C_1=\frac{\Sigma\,52}{EJ},$$

$$C_2=\frac{\Sigma_1}{EJ},\quad D_1=\frac{\Sigma\,53}{EJ}-\frac{\Sigma\,46}{EA}-\frac{\Sigma\,49}{GA} \text{ and } D_2=\frac{\Sigma\,60}{EJ}$$

In this manner, with the values N_A, T_A and M_A, using the equations (4), we can find the real axial and shear forces and the bending moments for all sections along the periphery of the frame, and consequently check its dimensions.

In the special case when the forces F_1 ..... F_r are not arbitrary, but symmetric with respect to the z-axis of symmetry of the cross section, T_A is equal to zero, the equation (2) vanishes, the shear-flows t_i are equal to t_i^T, and in the tables T 1 and T 3 the following ten columns are to be omitted: 16 to 19, 48 and 55 to 59. Besides, the tables are calculated only for half the section, i.e. up to the section n/2.

The sums $\Sigma\,1$, $\Sigma\,46$, $\Sigma\,47$, $\Sigma\,49$, $\Sigma\,50$, $\Sigma\,52$, $\Sigma\,53$, $\Sigma\,54$ and $\Sigma\,60$ in all earlier deduced equations can be obtained by multiplying by 2 the sums obtained by summations up to the section n/2.

T-1

COLUMN →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	λ_i	$A_{ik} = \lambda_i \delta_i$	A_{is}	ζ_{ik}	ζ_{is}	$A_{ik} \cdot \zeta_{ik}$	$A_{is} \cdot \zeta_{is}$	$Z_{ik} = \zeta_{ik} - \zeta_0$	$Z_{is} = \zeta_{is} - \zeta_0$	$A_{ik} \cdot Z_{ik}^2$	$A_{is} \cdot Z_{is}^2$	$A_{ik} \cdot Z_{ik}$	$A_{is} \cdot Z_{is}$	$(S_y)_i$	$t_i^T = \frac{\Delta T}{J_y} \cdot 14$	r_i	$2\Delta A_i = \lambda_i r_i$	$t_i^M = \frac{\Delta M_t}{2A}$	$t_i = t_i^T + t_i^M$	$t_i^m = \frac{t_{i-1} + t_i}{2}$
1																				
2																				
⋮																				
$i-1$																				
i																				
$i+1$																				
⋮																				
$\frac{n}{2}-1$																				
$\frac{n}{2}$																				
		Σ2	Σ3			Σ6	Σ7			Σ10	Σ11						Σ17			
$\frac{n}{2}+1$																				
⋮																				
$n-1$																				
n																				
	Σ1																			

T-2

COLUMN →	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
	$\Delta F_i = t_i^m \cdot \lambda_i$	α_i	$\sin \alpha_i$	$\cos \alpha_i$	$\Delta F_{yi} = -\Delta F_i \cos\alpha_i$	$\Delta F_{zi} = -\Delta F_i \sin\alpha_i$	$a_i = \sum_{\nu=1}^{\nu=i} \Delta F_{y\nu}$	$b_i = \sum_{\nu=1}^{\nu=i} \Delta F_{z\nu}$	$\sum_{k=1}^{q} F_k$	$a_i \cos\alpha_i$	$b_i \sin\alpha_i$	$\sin\alpha_i \sum_{k=1}^{q} F_k$	$a_i \sin\alpha_i$	$b_i \cos\alpha_i$	$\cos\alpha_i \sum_{k=1}^{q} F_k$	$N_{0i} =$ 30 + 31 + 32	$T_{0i} =$ 33 - 34 - 35	y_i	z_i	$z_{i-1} - z_i$
1																				
2																				
⋮																				
$i-1$																				
i																				
$i+1$																				
⋮																				
$\frac{n}{2}-1$																				
$\frac{n}{2}$																				
$\frac{n}{2}+1$																				
⋮																				
$n-1$																				
n																				

COLUMN →	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
	$y_i - y_{i-1}$	$\sum_{k=p}^{q} F_k(y_i - y_k)$	$c_i = 40 \cdot a_{i-1}$	$d_i = 41 \cdot b_{i-1}$	$M_{oi} = M_{o(i-1)} - 43 - 44 - 42 - 41 \sum_{k=1}^{i-1} F_k$	$\lambda_i \cdot N_{oi} \cdot \cos\alpha_i = 1 \cdot 36 \cdot 24$	$\lambda_i \cdot \cos^2\alpha_i = 1 \cdot 24^2$	$\lambda_i \cos\alpha_i \cdot \sin\alpha_i = 1 \cdot 24 \cdot 23$	$\lambda_i \cdot T_{oi} \cdot \sin\alpha_i = 1 \cdot 37 \cdot 23$	$\lambda_i \cdot \sin^2\alpha_i = 1 \cdot 23^2$	$e_1 - z_i = e_1 - 39$	$(e_1 - z_i) \cdot \lambda_i = 1 \cdot 51$	$\lambda_i \cdot M_{oi}(e_1 - z_i) = 52 \cdot 45$	$(e_1 - z_i)^2 \cdot \lambda_i = 1 \cdot 51^2$	$\lambda_i \cdot N_{oi} \cdot \sin\alpha_i = 1 \cdot 36 \cdot 23$	$\lambda_i \cdot T_{oi} \cdot \cos\alpha_i = 1 \cdot 37 \cdot 24$	$\lambda_i y_i = 1 \cdot 38$	$\lambda_i \cdot y_i \cdot M_{oi} = 57 \cdot 45$	$\lambda_i y_i^2 = 57 \cdot 38^2$	$M_{oi} \cdot \lambda_i = 1 \cdot 45$
1																				
2																				
⋮																				
$i-1$																				
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$\frac{n}{2}$																				
$\frac{n}{2}+1$																				
⋮																				
$n-1$																				
n																				
						$\Sigma 46$	$\Sigma 47$	$\Sigma 48$	$\Sigma 49$	$\Sigma 50$		$\Sigma 52$	$\Sigma 53$	$\Sigma 54$	$\Sigma 55$	$\Sigma 56$	$\Sigma 57$	$\Sigma 58$	$\Sigma 59$	$\Sigma 60$