On the dynamic longitudinal stability of sailplanes with fixed and free controls *

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Equations

The longitudinal motion of an airplane referred to eulerian wind axes (fig. 1), in the "elevator free" condition, is defined by the following set of differential equations:

$$(1)\quad \left\{ \begin{aligned} F_x &= m\dot{V} \\ F_z &= -mV\dot{\gamma} = -mV(\dot{\vartheta} - \dot{a}) \\ M &= J_y\ddot{\vartheta} \\ HM &= J_e\ddot{\delta} \end{aligned} \right.$$

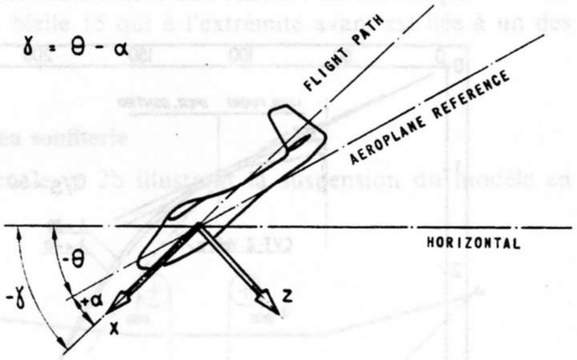

Fig. 1

where:

F_x, F_z = components, along X and Z axes, of the resultant external forces

M = moment of the external forces about the Y axis (passing through the airplane C.G., perpendicular to XZ plane)

m = mass of the airplane

V = airplane velocity along the flight path

J_y = airplane moment of inertia about Y axis

HM = elevator hinge moment

δ = elevator deflection

J_e = elevator moment of inertia about the hinge axis (it should include the inertia of the control transmission)

In the "elevator fixed" condition, only the first three parts of equation (1) occur.

The equations are elaborated for ease of solution.

The major variables to be considered are the following:

- forward speed, V
- angle of attack, a
- attitude angle, ϑ
- elevator deflection, δ

Forces and moments are expressed as a function of the above variables.

The following basic assumptions are made:

- it is assumed that the total change in the forces and moments is made up of the partial derivatives of the forces and moments taken with respect to each of the major variables and with the rate of change of these variables with time.
- it is assumed that the displacements of each of the variables from the equilibrium conditions are small, the partial derivatives (stability derivatives) can be considered constants and the differentials (dV, da, $d\vartheta$, $d\delta$) replaced by the actual increments (ΔV, Δa, $\Delta\vartheta$, $\Delta\delta$).

The equations are simplified by eliminating all terms that clearly do not appreciably affect the motion, and non-dimensionalised in the classical way [ref. 1, 2, 3]. They are finally written as follows:

$$(2)\quad \left| \begin{aligned} &\left(C_D + \frac{d}{d\tau}\right)u + \frac{1}{2}\left(C_{Da} - C_L\right)\Delta a + \frac{1}{2}C_L\,\Delta\vartheta = 0 \\ &C_L u + \left(\frac{1}{2}C_{La} + \frac{d}{d\tau}\right)\Delta a - \frac{d}{d\tau}\Delta\vartheta = 0 \\ &\left(C_{ma} + C_{mda}\frac{d}{d\tau}\right)\Delta a + \left(C_{md\vartheta}\frac{d}{d\tau} - h\frac{d^2}{d\tau^2}\right)\Delta\vartheta + \\ &+ \left(C_{m\delta} + C_{md\delta}\frac{d}{d\tau}\right)\delta = 0 \\ &\left(C_{ha} - h_1\frac{d}{d\tau}\right)\Delta a + \left(C_{hd\vartheta}\frac{d}{d\tau} - h_2\frac{d^2}{d\tau^2} + h_1\frac{d}{d\tau} - \right. \\ &\left. - l_1\frac{d^2}{d\tau^2}\right)\Delta\vartheta + \left(C_{h\delta} + C_{hd\delta}\frac{d}{d\tau} - h_2\frac{d^2}{d\tau^2}\right)\delta = 0 \end{aligned} \right.$$

where:

$u = \Delta V/V$, $\tau = t/t'$ (aerodynamic time), $t' = m/\rho SV$

The definition of symbols relating to the stability derivatives and coefficients is given in the next paragraph.

Solution

The data relating to a sailplane of the "Standard Class", as defined by the OSTIV-FAI specifications [ref. 4], have been entered in the equations.

In the great variety of existing sailplanes, the weights and the geometric characteristics of "standard" sailplanes may be considered to represent an average. It is thought therefore that the conclusions drawn from the results of the calculations may apply to a considerable number of existing sailplanes.

The main data relating to the chosen sailplane are:

b = wing span = 15 m

S = wing surface = 13.1 m²

A = aspect ratio = 17.1

c_{av} = wing geometric mean chord = 0.88 m

W = total weight = 300 kg

K_y = sailplane's radius of gyration about the Y axis = 1.84 m (evaluated theoretically)

l_t = distance from the sailplane's center of gravity to the elevator hinge axis = 3.74 m

μ_e = $m_e/\rho S_e c_e = 7.46$ (m_e = mass of the elevator = 0.204 kg $m\,s$. S_e = elevator surface = 0.81 m², c_e = elevator mean chord = 0.27 m)

x_e = distance from the elevator center of gravity to the elevator hinge axis = 0.13 m

k_e = elevator's radius of gyration about the hinge axis = 0.164 m (evaluated theoretically)

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* This paper was presented at the 8th OSTIV Congress, Cologne, June 1960, in a shorter version, covering only the "free controls" case. The present, more extended version was presented at the 4th European Aeronautical Congress, Cologne, 1960, and is also published in the "Jahrbuch 1960 der WGL".

Two basic configurations of the sailplane have been considered:

—with airbrakes retracted
—with airbrakes fully extended

The airbrakes, in this particular case, are of the segmented surface type, and are located on both the upper and lower wing surfaces, at about 40% of wing chords.

The speed polars of the sailplane, and hence the C_L, C_D polars, are known from flight tests. They, however, have been approximated by quadratic equations, in the C_L interval which is of interest in this particular case ($C_L = 0.5$ to 1.2).

Since the airbrakes have a considerable effect on the span lift distribution, $C_{L\alpha} = dC_L/d\alpha$ and $C_{D\alpha} = dC_D/d\alpha$ have been calculated for both configurations.

Calculated values of the coefficients and of the stability derivatives are as follows:

$$C_D = \begin{cases} 0.01 + 0.02\,C_L^2 \text{ (airbrakes retracted)} \\ 0.131 + 0.031\,C_L^2 \text{ (airbrakes extended)} \end{cases}$$

$$C_{L\alpha} = dC_L/d\alpha = \begin{cases} 5.39 \text{ (airbrakes retracted)} \\ 5.03 \text{ (airbrakes extended)} \end{cases}$$

$$C_{D\alpha} = dC_D/d\alpha = \begin{cases} 0.216\,C_L \text{ (airbrakes retracted)} \\ 0.312\,C_L \text{ (airbrakes extended)} \end{cases}$$

$$C_{md\alpha} = \frac{dC_m}{d\left(\frac{d\alpha}{d\tau}\right)} = -0.1048 \qquad C_{md\vartheta} = \frac{dC_m}{d\left(\frac{d\vartheta}{d\tau}\right)} = -0.461$$

$$C_{m\delta} = \frac{dC_m}{d\delta} = -1.33 \qquad C_{md\delta} = \frac{dC_m}{d\left(\frac{d\delta}{d\tau}\right)} = -0.03025$$

$$C_{h\alpha} = \frac{dC_n}{d\alpha} = -0.533 \qquad C_{hd\vartheta} = \frac{dC_n}{d\left(\frac{d\vartheta}{d\tau}\right)} = -0.1427$$

$$C_{h\delta} = \frac{dC_n}{d\delta} = -0.95 \qquad C_{hd\delta} = \frac{dC_n}{d\left(\frac{d\delta}{d\tau}\right)} = -0.0344$$

(C_h = elevator hinge moment coefficient = $2\,HM/S_e c_e \rho V^2$)

$$\mu = \frac{W/g}{\rho S c_{av}} = 19.8 \qquad h = \frac{2\,k_y^2}{\mu c_{av}^2} = 0.388$$

$$h_1 = \frac{2\,\mu_e x_e}{\mu c_{av}} = 0.104 \qquad h_2 = \frac{2\,\mu_e x_e^2}{\mu^2 c_{av}^2} = 0.001165$$

$$l_1 = \frac{2\,\mu_e x_e l_t}{\mu^2 c_{av}^2}$$

The calculations have been made at various C.G. longitudinal locations, i.e. for various values of $C_{m\alpha} = dC_m/d\alpha$:

$C_{m\alpha}$	0	–0.3	–0.7	–0.75	–0.8	–1	–1.5	–2
C.G. location % m.a.c.	48	42.5	35	34	33	30.4	20	9.6

The sailplane neutral point in the "stick fixed" condition is therefore at 48% m.a.c. In the "stick free" condition its location is at 34% m.a.c. ($C_{m\alpha} = -0.75$).

It is well known that the critical conditions, for what concerns the damping, occur at high C_L. The equations have been solved, therefore, for $C_L = 1$.

The above mentioned conditions are summarized in the following table:

$C_L = 1$			
"fixed controls"		"free controls"	
Air Brakes retracted	Air Brakes extended	Air Brakes retracted	Air Brakes extended
C.G. locations from 10% to 50% m.a.c. approximately			

The equations have been solved by the aid of an analogue computer ("Shorts").

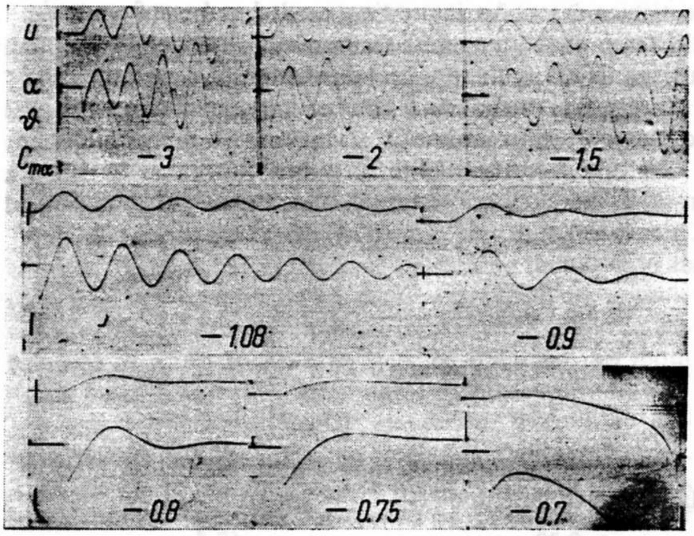
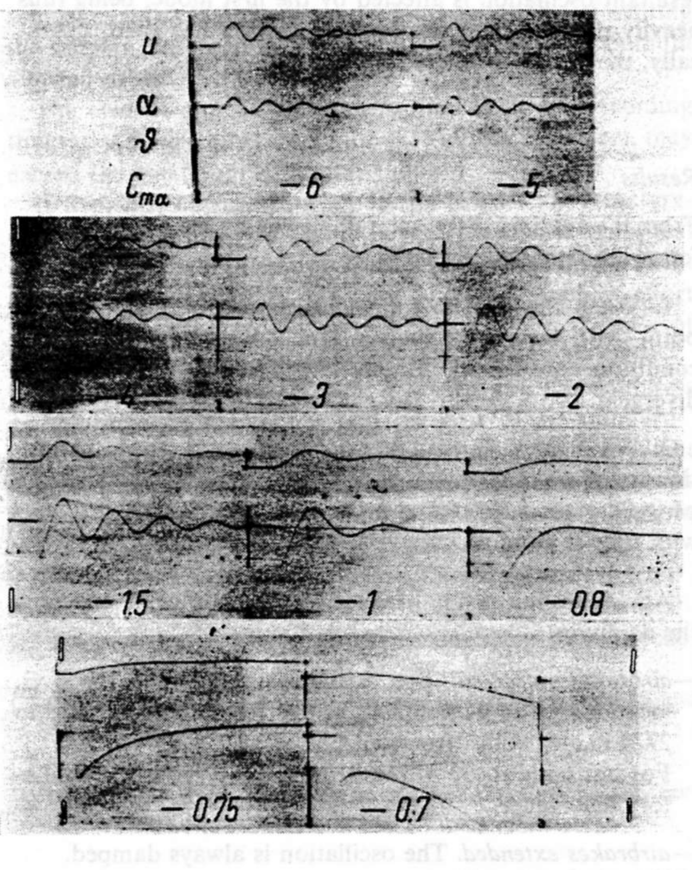


Fig. 2 Free Controls: Airbrakes retracted

Fig. 3 Free Controls: Airbrakes extended



The disturbance that originates the motion, the characteristics of which are to be determined here, has been introduced into the machine as an abrupt variation of the angle of attack α.

The solutions given by the computer have been recorded as diagrams of u, α, ϑ versus $\tau = t/t'$ (aerodynamic time).

Typical records are shown on fig. 2 and 3.

For each particular condition (fixed or free controls, airbrakes retracted or extended, $C_L = 1$) the machine gives one diagram $u(\tau)$ or $\alpha(\tau)$ or $\vartheta(\tau)$. This diagram is the sum of the two oscillatory modes that, as it is well known, represent, in general, separate and simultaneous solutions of eq. (2).

In order to determine the characteristics of the two oscillatory modes, eq. (2) have been solved analytically in some particular cases. An interesting result has been found: the difference of the period of the two oscillatory modes is not such as to justify the classical distinction of "phugoid" and "short period mode", as it is usual in the case of aircraft of considerably different size.

The two periods are of the same order of magnitude. For instance, with "fixed controls" and "airbrakes retracted", $C_L = 1$ and $C_m\alpha = -1.08$ (C.G. at about 30% m.a.c.), it has been found:

$p_1 \;=\; 4.3$ sec $\qquad p_2 \;=\; 13.1$ sec

In the same conditions, but with "airbrakes extended":

$p_1' \;=\; 8.5$ sec $\qquad p_2' \;=\; 12.8$ sec

The dampings, however, show a great difference. Time to damp to half amplitude is:

$s_1 \;=\; 0.32$ sec $\qquad s_2 \;=\; 67.7$ sec

$s_1' \;=\; 0.43$ sec $\qquad s_2' \;=\; 15.4$ sec

It is clear, therefore, that only the very first part of the resultant oscillation is affected by the first mode, being thus heavily damped. The resultant oscillation has then, practically, the period of the second mode.

Results

From the analysis of the solutions given by the computer, the following remarks arise:

1. When the sailplane C.G. is aft the sailplane neutral point (with fixed or free controls, according to the particular condition considered), the motion is always aperiodic and divergent.

2. When the C.G. is in front of the neutral point, the motion is always oscillatory. As the C.G. is shifted forward, the period becomes shorter and the rate of variation of u, α, ϑ with time is more accentuated.

An interesting result has been found for what concerns the damping:

—*airbrakes retracted.* The oscillation is damped for C.G. locations up to 34% m.a.c., with "fixed controls", up to 27% m.a.c., with "free controls".

For positions ahead of these, the motion shows an oscillatory divergence.

—*airbrakes extended.* The oscillation is always damped.

This is clearly illustrated in fig. 4, where the damping constant "a" * is plotted as a function of the C.G. location. The region of positive "a" is that of the unstable oscillations.

The period in sec is indicated on some points along the curves. It is evident that the airbrakes practically do not affect the period. They, however, affect heavily the damping of the motion and render it stable throughout the useable range of C.G. positions of the sailplane.

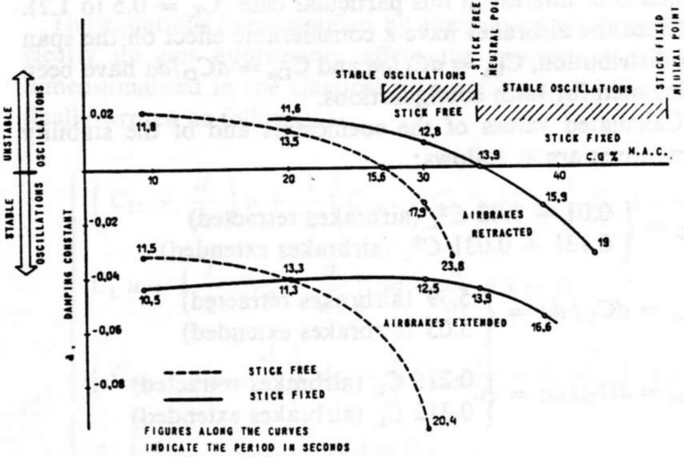


Fig. 4

3. The recordings show additionally that the opening of the airbrakes reduces the amplitude of the oscillation.

For instance, the upper diagram of fig. 5, shows u and α oscillations with free controls and C.G. at 30% m.a.c.: the oscillations are damped in both conditions (airbrakes retracted and extended); the amplitude, however, is considerably smaller in the latter case.

The lower diagram of fig. 5 compares u and α oscillations in the same conditions of the preceding case, but with C.G. at 20% m.a.c. The oscillation is increasing, if the airbrakes are retracted; it is damped, with airbrakes extended.

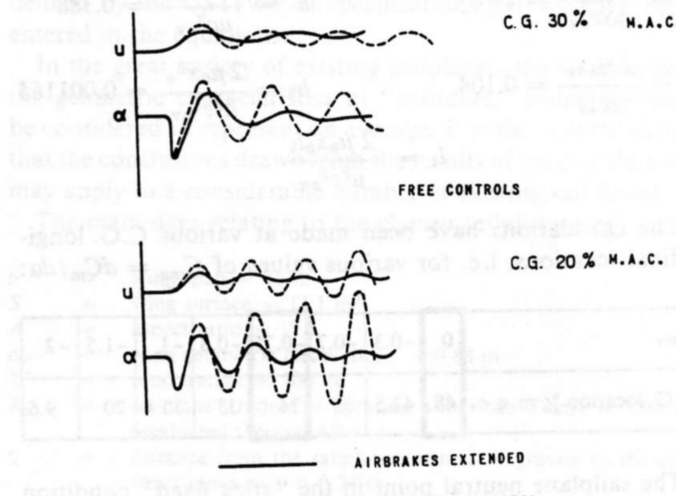


Fig. 5

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* The damping constant "a" is the real part of the roots $\lambda_{1,2} = a_{1,2} \pm i\, b_{1,2}$, of the stability quartic $\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0$. The coefficients of the quartic are a function of the stability derivatives and coefficients of eq. (2) (ref. 1-2-3). The damping constant "a" is correlated to the damping "s" defined as time to damp to half amplitude, by: $a = 0{,}693\ \tau/s$.

Practical meaning of the results

In the case of airplanes, the "phugoid" oscillation has not, generally, practical importance, and the attention is concentrated on the "short period" oscillation.

In the case of sailplanes, the two oscillatory modes should both be taken into consideration, owing to the particular mode of operation of this kind of aircraft.

It can be stated that the dynamic longitudinal behaviour of a sailplane has practical importance in two flight conditions: when flying in bumpy air and in "blind" flying.

In the case, very frequent indeed, when a sailplane gets in or out of an up or down current of air, the disturbance of the initial equilibrium conditions is the variation, more or less abrupt, of the angle of attack induced by the vertical current.

The sailplane's response to this disturbance is apparent to the pilot through the variation of the airspeed. Any glider pilot is, in fact, familiar with the increase of airspeed that occurs, for instance, when entering an up-current.

After a short time, the sailplane is brought back, by the pilot's manoeuvre, to the initial equilibrium conditions. It is clear, therefore, that only the very beginning of the perturbated motion takes place.

The calculations show how the sailplane's response to the angle of attack disturbance is an oscillation of u, α and ϑ; the more forward is the C.G., the stronger and wider will be the variations of u, α and ϑ in the initial part of the oscillation.

A suggestion to the sailplane designer comes straight from this remark: It is not convenient to have the C.G. too much forward, as it is sometimes desired with a view to a high static stability margin, if strong and wide airspeed variations when flying in bumpy air, are to be avoided. These oscillations, in fact, are uncomfortable to the pilot and require frequent or continuous action on the controls.

This trend of the pilot to correct, by his action on the controls, the deviation of the sailplane from the prefixed airspeed and attitude, is instinctive; in most cases he is not even aware of the static (and dynamic) instability that occurs when the C.G. is in an intermediate position between the "stick fixed" and "stick free" neutral points.

It is very different, however, in the case the pilot loses the control of the sailplane in blind flying. In this case he may not know the correct direction of movement of the controls in order to regain a normal attitude, and the sailplane may enter an oscillatory motion.

It is likely that this motion is something between the "stick free" and "stick fixed" case.

It is of evident importance in such a case that two requirements are fulfilled by the sailplane:

1. The oscillatory motion should be damped.

2. The amplitude of the airspeed oscillation should be within prefixed limits (not to exceed the maximum allowed airspeed).

The present study is a step toward a better understanding of the sailplane behaviour in the longitudinal oscillations. The study should be extended to cover also the asymmetric oscillatory motions. It does not seem illogical, however, to think that in the longitudinal motion the maximum airspeed variations may occur.

It is clear from the results of the calculations that, if a damped oscillation is wanted in the "airbrakes retracted" condition, a restriction is placed on the possible locations of the sailplane's C.G., as follows:

free controls: between the stick free neutral point and a well defined forward position. In the example: from 34% to 27% m.a.c.—see fig. 4.

fixed controls: between the stick fixed neutral point and a well defined forward position. In the example: from 48% to 34% m.a.c.—see fig. 4 (the forward position of the example accidentally corresponds to the stick free neutral point).

There exists, therefore, a forward limitation of the C.G. location: in the less restrictive case, it is that corresponding to the "stick free" condition. Ahead of it the oscillation is always unstable.

In the particular case considered, it is evident from fig. 4 that the two intervals of stable oscillations do not overlap. They are adjacent, and this indicates that there is no possibility of stable oscillations in both "stick free" and "fixed" conditions at the same time, the best compromise being for a single C.G. position (34% m.a.c.): in this case both "stick free" and "stick fixed" oscillations are neutrally damped.

It is evident, however, that an overlapping of the two intervals may be obtained in any case by shifting backwards the stick free neutral point. This may be obtained by suitable variations of the elevator characteristics (aerodynamic underbalance). A C.G. range may thus be obtained, for both "stick fixed" and "stick free" stable oscillations.

In the "airbrakes extended" condition, there is no forward limitation to the C.G.: the oscillation is always damped. A suggestion to the pilot may be logically derived from this consideration: the airbrakes should be extended whenever dangerous airspeed oscillations arise (as it may easily occur in cloud flying). In fact, a strong damping and a limitation of the amplitude of the oscillation is thus obtained.

An important OSTIV-FAI specification on "standard" sailplanes states that the airbrakes must be capable of limiting the sailplane airspeed in a vertical dive within the maximum allowed value.

The evident aim of this specification is that of avoiding that the sailplane airspeed, in the case of loss of control, may exceed the maximum structural airspeed.

It happens, however, owing to the usual stick force gradients, that, in order to maintain the sailplane in a vertical dive, it is necessary to apply a considerable effort on the stick, and it is difficult to think of a pilot who exerts such an effort in a forward direction, "airbrakes extended", in the case of loss of control of the sailplane.

The OSTIV-FAI specification is severe, and has doubtless an influence on the weight and the cost of a "standard" sailplane. This study, duly completed and extended, might suggest that the maximum attainable airspeed in a free oscillation, under convenient assumptions, is taken as a basis in the maximum airspeed specifications.

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