A Spherical Vortex Model of the Buoyant Thermal in Cumulus and Dry Convection[1,2]

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Within the last few years, laboratory experiments and observations have led to a spherical or ring-vortex conception of the buoyant element in atmospheric convection. Recently I have had an opportunity to witness laboratory experiments by Schaefer in which buoyant jets broke up into elements that looked very much like spherical vortices. Therefore turbulent convection probably does tend to consist of such vortex elements. Very likely even turbulent steady state jets consist of a conglomeration of overlapping vortex elements.

In my paper (1959), I combined Stommel's entrainment ideas (1947) with a spherical vortex model. Actually my adaptation of Stommel's ideas may be applied to any dynamic model. The spherical vortex was used only to obtain an approximate description of the internal motion of a bubble-like thermal and its effect on the surrounding air.

The problem may be treated very simply by divesting it of any explicit introduction of the spherical vortex aspect. The behavior of a thermal is grossly described in terms of equations for the conservation of mass, momentum, potential temperature, and specific humidity. The potential temperature and specific humidity conservation equations may easily be combined in an equation for the conservation of equivalent potential temperature.

Table of conservation equations

Mass:

$$\frac{dM}{dt} = \left(\frac{dM}{dt}\right)_{+} - \left(\frac{dM}{dt}\right)_{-} \tag{1}$$

Momentum:

$$\frac{dMW}{dt} = Mg\left[\frac{\rho_0 - \rho}{\rho} - \frac{\rho_1}{\rho}\right] - \left[\left(\frac{dM}{dt}\right)_{+} + \left(\frac{dM}{dt}\right)_{-}\right]W - \frac{1}{2}\, C_D S a^2 \rho\, W^2 \tag{2}$$

Specific humidity:

$$\frac{dMx}{dt} = x_0\left(\frac{dM}{dt}\right)_{+} - x\left(\frac{dM}{dt}\right)_{-} - M\left(\frac{dx}{dt}\right)_{c} \tag{3}$$

Potential temperature:

$$\frac{dM\Theta}{dt} = \Theta_0\left(\frac{dM}{dt}\right)_{+} - \Theta\left(\frac{dM}{dt}\right)_{-} + M\left(\frac{d\Theta}{dt}\right)_{c} \tag{4}$$

where M is the mass of the thermal, ρ the air density, ρ_l the liquid water density, x the specific humidity, a the radius (or some similar characteristic dimension) of the thermal, C_D the aerodynamic drag coefficient, and S a shape factor (such as π for a spherical thermal). The subscript 0 is used to denote variables in the thermal's immediate environment. The quantities

$$\left(\frac{dM}{dt}\right)_{+} \text{ and } \left(\frac{dM}{dt}\right)_{-}$$

indicate symbolically the instantaneous rate of addition of environment air to the thermal and the rate of removal of thermal air, respectively.

The above equations are exact in principal. Difficulties arise in defining the precise extent of the thermal element and in the quantitative definition of

$$\left(\frac{dM}{dt}\right)_{+} \text{ and } \left(\frac{dM}{dt}\right)_{-}$$

If there is any marked coherence of behavior of a given thermal element, the former may be reasonably well defined. The latter can be shown to have a rather finite range of variation.

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The lower limit of both

$$\left(\frac{dM}{dt}\right)_+ \text{ and } \left(\frac{dM}{dt}\right)_-$$

is zero for a thermal not mixing with the environment. The upper limit may be defined in terms of a complete exchange of mass when the thermal moves through a distance of order a. If we imagine the surface of a thermal to behave like a wire mesh through which the air can flow unimpeded, the upper limit becomes quite definite and intelligible. The drag coefficient C_D likewise has definite upper and lower limits. Thus by introducing in addition to C_D the arbitrary parameters K_1 and K_2 defined by

$$\frac{1}{M}\left(\frac{dM}{dt}\right)_- = K_1 \frac{W}{a} \qquad (5)$$

and

$$\frac{1}{M}\left(\frac{dM}{dt}\right)_+ = K_2 \frac{W}{a} \qquad (6)$$

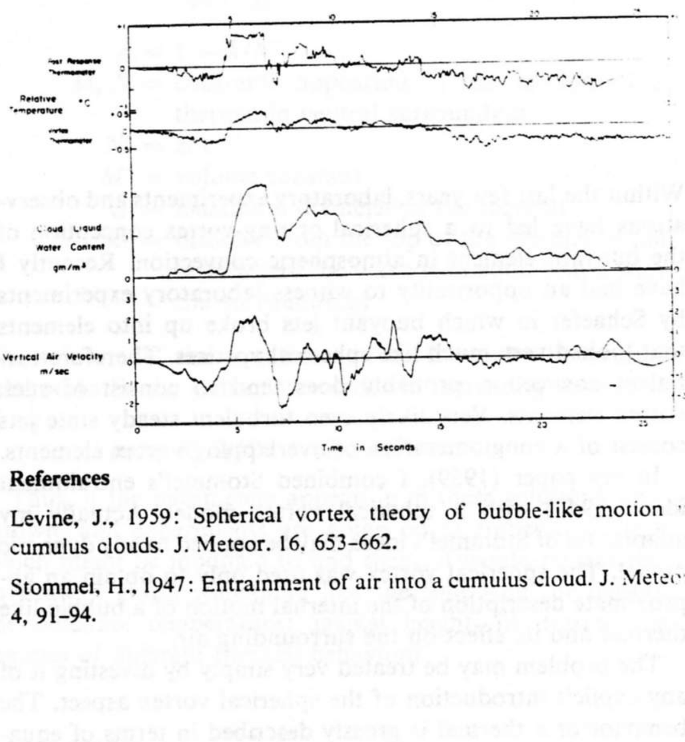
the problem of determining the motion of a thermal of given size in a given environment is completely determined by the conservation equations (1) to (4) and the knowledge of the saturation specific humidity as a function of potential temperature and pressure contained in thermodynamic charts except for the arbitrariness of the parameters K_1, K_2, C_D. Computations of this sort have been made and are illustrated in my paper (1959).

The behavior of the arbitrary parameters K_1, K_2, and C_D into which I have concentrated our ignorance may be possibly further limited by some semi-quantitative argument. However these parameters may be empirically evaluated from actual observations.

The above records were obtained by a DC-3 aircraft making a pass through a cloud tower at about 5000′ with cloud base at about 2000′ and with an indicated air speed of 100 knots in the neighborhood of Trinidad. The zero lines represent values just before entering cloud as references. Note the definitely buoyant thermal centered at 5.8 secs and approximately 2 secs in extent (100 meters). Also note the relatively cool portion just to the left of the thermal and towards the end of the pass. There are distinct maxima in liquid water content and vertical velocity coinciding with the above-mentioned thermal. The vertical velocity profile of the thermal appears remarkably similar to that of a spherical vortex.

Temperature, liquid water content, and vertical velocity measurements from aircraft flights through cumulus appear to indicate that the major portion of visible cloud consists of turbulent air cool with respect to the environment and relatively small positively buoyant elements. The positively buoyant elements apparently are consistently quite uniform and relatively high in liquid water content and temperature with vertical velocity profiles very much like that of a spherical vortex. But it must be realized that an elongated thermal or jet might have the same sort of velocity profile.

Eventually I hope to determine the behavior of K_1, K_2, and C_D by working backwards from the observations of temperature, cloud liquid water content, and vertical velocity in buoyant thermals. The relatively high liquid water contents of buoyant thermals lead me to believe that radar might be a very useful additional tool for exploring the extent and shape of buoyant thermals in clouds.



References

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