All-Moving Tailplanes

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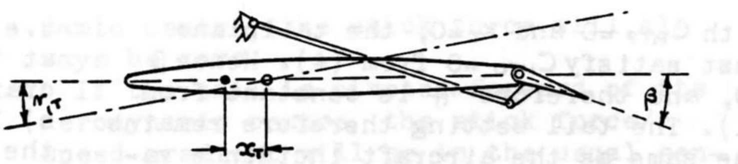

1. Summary

This note is intended to summarize the more important properties of all-moving tails fitted with gear tabs. Equations are quoted which describe their more important features. In particular, it is shown that by suitable choice of the tail hinge axis and other parameters the stick-free neutral point may be located anywhere between the aerodynamic centre of the aircraft-less-tail and the a.c. of the tail. There should be little difficulty in achieving satisfactory stick-free characteristics by suitable choice of hinge axis position, tab size and tab gearing.

2. Introduction

All-moving tailplanes are not particularly new: they were used, without geared tabs, on various pre-war gliders, and since the war, with tabs, on the Eon Olympia 419 and various light aircraft. This paper makes no claim to originality, but merely gathers together some interesting data which displays the properties of such an arrangement. No attempt has been made to show the derivation of the various equations in detail: the procedure is quite straightforward and conventional, although often lengthy. Rigid aircraft are assumed throughout and the static margin is therefore assumed to be equal to the C.G. margin.

It is assumed that no mechanical moments due to weights or springs are applied to the control system.

3. Assumptions

The tailplane (assumed to be of basically symmetrical section) rotates relative to the fuselage about a hinge line, which lies in the plane of its chord lines and is located a distance x_T aft of its aerodynamic centre. It is fitted with a geared tab which deflects in the same sense as the tailplane so that

$$\beta = \beta_0 + k\eta_T \tag{1}$$

where β_0 is a datum setting adjustable for trimming purposes and k is a positive gear ratio.

When the tab is deflected, it produces a pitching moment about the tail aerodynamic centre such that

$$C_{MTO} = C_3\beta \tag{2}$$

In the usual fashion, the tail lift coefficient is

$$C_{LT} = a_1\alpha_T + a_3\beta \tag{3}$$

and hence the tail hinge moment coefficient becomes

$$C_{MTP} = C_{MTO} + C_{LT}\frac{x_T}{C_T} \tag{4}$$

It is also assumed that

$$\alpha_T = \alpha - \varepsilon + \eta_T$$
$$= \alpha\left(1 - \frac{\delta\varepsilon}{\delta\alpha}\right) + \eta_T \tag{5}$$

if the downwash at $\alpha = 0$ is neglected.

4. Stick- fixed Stability

Stick-fixed, the longitudinal stability is, of course, the same as that of an aircraft with a conventional tail of the same size. In particular, the C.G. margin is given by:

$$H_n = (h_0 - h) + V_T\frac{a_1}{a}\left(1 - \frac{\delta\varepsilon}{\delta\alpha}\right) \tag{6}$$

where

$$V_T = \frac{\bar{V}'}{1+F} \tag{7}$$

and

$$F = \frac{a_1}{a}\frac{S_T}{S}\left(1 - \frac{\delta\varepsilon}{\delta\alpha}\right) \tag{8}$$

It also follows that the pitching moment coefficient of the aircraft about its C.G. is

$$C_{MG} = C_{M0} - C_L H_n - V_T\left[(a_1 + ka_3)\eta_T + a_3\beta_0\right] \tag{9}$$

and hence the tailplane setting to trim is

$$(\eta_T)_0 = \frac{1}{a_1 + ka_3}\left[\frac{C_{M0} - C_L H_n}{V_T} - a_3\beta_0\right] \tag{10}$$

or

$$\frac{\delta(\eta_T)_0}{\delta C_L} = -\frac{1}{a_1 + ka_3}\frac{H_n}{V_T} \tag{11}$$

i.e. $(a_1 + ka_3)$ replaces a_1 in the corresponding expression for a conventional tail.

5. Stick-free Tail Lift Curve Slope

The significant properties of all-moving tails are associated with the stick-free case.

It follows from the foregoing equations that if $C_{MTP} = 0$, the effective tail lift curve slope is given by:

$$\bar{a}_1 = \frac{1}{1 - \frac{\delta\varepsilon}{\delta\alpha}} \cdot \frac{\delta C_{LT}}{\delta\alpha} = a_1\left[\frac{kc_3}{kc_3 + (a_1 + ka_3)\frac{x_T}{c_T}}\right] \tag{12}$$

Hence:

a) If $x_T = 0$, $a_1 = \bar{a}_1$ and the stick-fixed and stick-free C.G. margins are equal. This can be seen more simply as follows:

With $C_{MTP}=0$ and $x_T=0$, the tailplane must satisfy $C_{MTO}=0$ from (4). Hence $\beta=0$, and therefore η_T is constant from (1). The tail setting therefore remains the same as the aircraft incidence varies. This result is independent of tab size, which will therefore be decided by stick-force considerations.

b) If $kc_3=0$ (i.e. if no tab is fitted), $\bar{a}_1=0$. The tail therefore makes no contribution to the stick-free stability, and the aircraft will only be stable if the C.G. is ahead of the wing-plus-fuselage aerodynamic centre. Again, this result is obvious on physical grounds: in the absence of a tab, the tail will trail along the local airflow direction and hence the tail lift coefficient will always be zero. In practice, it would also have to satisfy the condition

$$\frac{\delta C_{MTP}}{\delta C_{LT}}<0$$

i.e. the hinge line would have to be ahead of the tail aerodynamic centre.

c) If $x_T<0$ (i.e. hinge line ahead of a.c.), $\bar{a}_1<0$, and the stick-free C.G. margin will be less than the stick-fixed C.G. margin.

If $x_T>0$ (i.e. hinge line aft of a.c.), $\bar{a}_1>a_1$, and the converse of the above will be true.

In particular as $\frac{x_T}{c_T}\rightarrow -\frac{kc_3}{a_1+ka_3}$

(remembering that C_3 is negative)

then $\bar{a}_1\rightarrow\infty$

This condition sets a rearward limit to the location of the hinge line. If x_T is equal to the limiting value given above, any disturbance in pitch would produce an indefinitely large displacement of the tail.

It is therefore clear that, for a given stick-fixed C.G. margin, the stick-free C.G. margin can be varied within very wide limits by adjusting the tab size, gear ratio, and tail plane hinge line.

6. Stick-free C.G. Margin

The expression for H_n' is similar to equation (6), but with $\bar{a}_1$ replacing a_1, viz.:

$$H_n'=(h_0-h)+\bar{V}_T\frac{\bar{a}_1}{a}\left(1-\frac{\delta\varepsilon}{\delta\alpha}\right) \qquad (13)$$

where

$$\bar{V}_T=\frac{1}{1+\bar{F}} \qquad (14)$$

and

$$\bar{F}=\frac{\bar{a}_1}{a}\,\frac{S_T}{S}\left(1-\frac{\delta\varepsilon}{\delta\alpha}\right) \qquad (15)$$

Hence:

a) As mentionned above, if $x_T=0$, $\bar{a}_1=a_1$ and then $H_n'=H_n$

b) If $kc_3=0$, $\bar{a}_1=0$ and $H_n'=(h_0-h)$ or $h_n'=h_0$

i.e. the neutral point, stick-free, is at the aerodynamic centre of the aircraft-less-tail.

c) It is also interesting to consider the case in which $\bar{a}_1\rightarrow\infty$ (see para 5 (c) above).

(13) may also be written

$$H_n'=(h_0-h)+\frac{\bar{V}'}{1+\bar{F}}\cdot\bar{F}\cdot\frac{S}{S_T} \qquad (16)$$

As $\bar{a}_1\rightarrow\infty$, $\bar{F}\rightarrow\infty$ and hence

$$H_n'\rightarrow(h_0-h)+\bar{V}'\frac{S}{S_T}$$

i.e.

$$H_n'\rightarrow(h_0-h)+\frac{l_T}{\bar{c}}=H_{n\infty}' \qquad (17)$$

or

$$h_n'\rightarrow h_0+\frac{l_T}{\bar{c}} \qquad (17a)$$

so that as the tail hinge line tends towards its rearmost limiting position, the stick-free neutral point approaches the aerodynamic centre of the tail.

7. Stick Force to Change Speed

The tail hinge moment coefficient is given by:

$$C_{MPT}=-\left(\frac{kc_3}{a_1+ka_3}\right)\left(\frac{a_1}{\bar{a}_1}\right)\left(\frac{H_n'}{\bar{V}_T}\right)C_L + \text{const.} \qquad (18)$$

Since C_3 is negative, it follows that if H_n' is positive,

$\frac{\delta C_{MPT}}{\delta C_L}$ is positive,

i.e. the pilot has to pull to increase C_L, as is usual.

a) It is of interest to consider how $\frac{H_n'}{\bar{a}_1\bar{V}_T}$ varies with $\bar{a}_1$

This quantity may be written

$$\left(\frac{h_0-h}{\bar{V}'}\right)\left(\frac{1}{\bar{a}_1}+\frac{\bar{F}}{\bar{a}_1}\right)+\frac{1}{a}\left(1-\frac{\delta\varepsilon}{\delta\alpha}\right)$$

and hence if h is fixed, it tends to the value

$$\frac{1}{a}\left(1-\frac{\delta\varepsilon}{\delta\alpha}\right)\frac{\bar{c}}{l_T}\left(h_0-h+\frac{l_T}{\bar{c}}\right)$$

i.e.

$$\frac{1}{a}\left(1-\frac{\delta\varepsilon}{\delta\alpha}\right)\frac{\bar{c}}{l_T}H_{n\infty}' \quad \text{as } \bar{a}_1\rightarrow\infty$$

so that even when the effective tail lift curve slope, stick-free, has become infinite, the quantity

$\frac{\delta C_{MPT}}{\delta C_L}$ remains finite.

This quantity is, of course, simply related to the stick-force to change speed.

b) If $x_T=0$, $a_1=\bar{a}_1$ and then

$$\frac{\delta C_{MPT}}{\delta C_L}=-\left(\frac{kc_3}{a_1+ka_3}\right)\frac{H_n}{V_T} \qquad (19)$$

c) If no tab is fitted, equation (18) simplifies to give:

$$\frac{\delta C_{MPT}}{\delta C_L}=\frac{x_T}{c_T}\left(\frac{h-h_0}{\bar{V}'}\right) \qquad (20)$$

If $h > h_o$, as is usual, then this quantity is positive when x_T is positive (i.e. the hinge line is ahead of the tail aerodynamic centre). However, in such a case

$$\frac{\delta C_{MPT}}{\delta \eta_T}$$ would be positive

and the tail itself would be unstable.

The only satisfactory arrangement is to have a positive stick-free static margin (in this case $h < h_o$) and make x_T negative (i.e. hinge line of tail ahead of its aerodynamic centre).

d) It follows from (18) that the stick force at V_i is given by:

$$P_e = -\frac{M_e S_T C_T W}{S}\left(\frac{kc_3}{a_1+ka_3}\right)\left(\frac{a_1}{\bar{a}_1}\right)\left(\frac{H_n'}{V_T}\right)\left(1-\frac{V_i^2}{V_{io}^2}\right) \quad (21)$$

where V_{io} is the speed at which the aircraft is trimmed (i.e. $P_e = 0$).

This is of the same form as the expression for an aircraft with a normal tail, and leads to the result that, at the trimmed speed V_{io}, the stick-force/speed gradient is

$$\left(\frac{\delta P_e}{\delta V_i}\right)_o = \frac{2 M_e S_T C_T W}{S}\left(\frac{kc_3}{a_1+ka_3}\right)\left(\frac{a_1}{\bar{a}_1}\right) \cdot \left(\frac{H_n'}{V_T}\right)\frac{1}{V_{io}} \quad (22)$$

8. Conclusions

a) The stick-fixed C.G. margin is, naturally, unaffected by the replacement of a conventional tail by an all-moving one of the same size.

b) The tailplane angle/C_L gradient (equation 11) is analogous to the usual expression for elevator angle/C_L gradient, a_2 being replaced by $(a_1 + ka_3)$.

c) If the tail is pivoted at its aerodynamic centre, the stick-fixed and stick-free C.G. margins are equal.

d) As the tail hinge line is moved aft of its aerodynamic centre, its effective lift-curve slope, stick-free, increases indefinitely. The aftmost limit of the hinge line is defined in para. 5 (c).

e) As the hinge line is moved from the tail aerodynamic centre to the limiting position, the stick-free neutral point moves from the stick-fixed neutral point to the tail aerodynamic centre.

f) The stick force to change speed remains finite even with the hinge line at its aftmost limit.

g) If no tab is fitted, the tail makes no contribution to the stick-free stability. Hence, for stability, the C.C. must be ahead of the aerodynamic centre of the aircraft-less-tail.

If the tail is pivoted at its aerodynamic centre, the stick force will always be zero.

If the tail is pivoted forward of its aerodynamic centre, the stick force/speed gradient will be in the usual sense but, in the absence of the tab there will generally be no speed at which the stick force is zero.

h) It is therefore clear that, for a given C.G. margin, stick-fixed, the stick-free C.G. margin and other associated quantities can be varied within extremely wide limits by suitable choice of tail hinge line position, tab size, and tab gearing.

9. List of Symbols

The standard British convention is used throughout.

$a = \frac{\delta C_{LW}}{\delta \alpha}$ for the aircraft-less tail

$a_1 = \frac{\delta C_{LT}}{\delta \alpha_T}$ tail lift curve slope, stick fixed

$\bar{a}_1 =$ effective tail lift curve slope, stick-free

$a_3 = \frac{\delta C_{LT}}{\delta \beta}$

$C_L =$ lift coefficient of complete aircraft

$C_{LW} =$ lift coefficient of aircraft-less-tail

$C_{LT} =$ lift coefficient of tail

$C_{Mo} =$ pitching moment coefficient of aircraft-less-tail about its aerodynamic centre

$C_{MTo} =$ moment coefficient of tail about its a.c.

$C_{MTP} =$ moment coefficient of tail about its hinge line

$\bar{C} =$ wing mean chord

$C_T =$ tail mean chord

$C_3 = \frac{\delta C_{MTO}}{\delta \beta}$

F see equation 8

$\bar{F}$ see equation 15

$H_n =$ stick-fixed C.G. margin

$H_n' =$ stick-free C.G. margin

$h_n\bar{C} =$ stick-fixed neutral point position aft of datum

$h_n'\bar{C} =$ stick-free neutral point position aft of datum

$h\bar{C} =$ C.G. position aft of datum

$h_o\bar{C} =$ aerodynamic centre position of aircraft-less-tail aft of datum

$H =$ tail hinge moment $= C_{MPT}\frac{1}{2}\rho_o V_i^2 S_T C_T$

$k =$ tab gear ratio

$\ell_T =$ tail moment arm (a.c. of aircraft-less-tail to a.c. of tail)

$M_e =$ stick-tailplane gear ratio such that $P_e = M_e H$

P_e = stick force
S = wing area
S_T = tail area
$\overline{V}'$ = tail volume coefficient $\frac{S_T \ell_T}{S\bar{c}}$
$V_T = \frac{V'}{1+F}$
$\overline{V}_T = \frac{\overline{V}'}{1+\overline{F}}$
V_i = equivalent airspeed
V_{io} = equivalent airspeed for H=0
W = all-up weight
x_T = distance of tail hinge line aft of tail a.c.
α = no-lift-line incidence of aircraft-less-tail
α_T = tail incidence
β = tab angle
β_o = tab datum setting
ε = downwash angle at tail
η_T = tail setting relative to aircraft-less-tail no-lift-line
$(\eta_T)_o$ = tail setting to trim
ρ_o = sea-level standard air density

(Swiss Aero Revue 1963/7)