The Optimum Size of Horizontal Tail for a Glider

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1. Summary

Calculations have been made to determine the optimum size of horizontal tail, from the performance point of view, for a typical Standard-Class glider. It was found that for a stick-fixed C.G. margin of 0.1, the optimum tail volume coefficient was about 0.45, but quite large variations in tail size had relatively little effect. When the stick-fixed C.G. margin was zero, the best performance was obtained with the smallest tail volume coefficient considered.

Although it is imprudent to generalize on the basis of a few calculations, these results suggest that, for optimum performance, a glider should have the minimum acceptable stick-fixed C.G. margin and a small tailplane. In practice this conclusion may have to be modified in order to obtain satisfactory dynamic characteristics.

2. Introduction

It was remarked at the 1960 World Gliding Championships that British gliders seemed to have rather large horizontal tails compared with many foreign machines. Tail volume coefficient quoted in Ref. 1 confirm this impression, some values being as follows:

Skylark 3F . .	0.712	Ka-6	0.445
Skylark 2B . . .	0.68	Elfe M	0.645
Olympia 419 . .	0.64	Meteor	0.443
Phönix . . .	0.531		

A further study of typical machines shows that the Skylark 3F had the largest value of those listed, the most common figure being about 0.5.

3. Physical Considerations

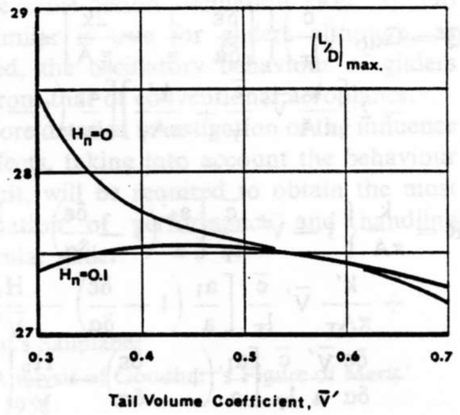
Unless the tail volume coefficient of a glider is very large, the tail load will be downwards (i.e. negative) under most operating conditions, assuming fairly conventional characteristics. If the tail size is increased, the C.G. may be moved further aft for a given stick-fixed C.G. margin. The tail load then becomes less negative at a given flight condition, and the associated induced drag is decreased. On the other hand, the tail profile drag is increased.

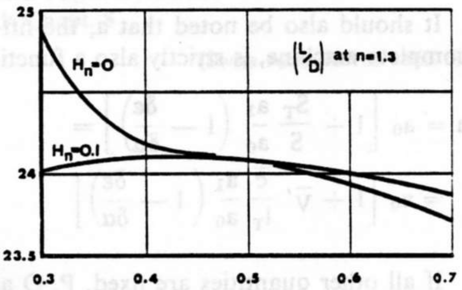
One would expect that, to some extent, these changes would be self-cancelling, but it is clearly worth investigating the problem to see whether there is an optimum tail size.

4. Analysis (For notation see paragraph 5)

If the total lift coefficient of the glider is C_L, the wing lift coefficient is given by:

$$C_{LW} = C_L - \frac{S_T}{S} C_{LT} \tag{1}$$





and the tail lift coefficient by

$$C_{LT} = \frac{C_{MO}}{\bar{V}'} + \left[\frac{a_1}{a}\left(1 - \frac{\delta\varepsilon}{\delta\alpha}\right) - \frac{H_n}{\bar{V}'}\right] C_L \tag{2}$$

The induced drag coefficient of the wing will be

$$C_{DiW} = \frac{kC_{LW}^2}{\pi A} \tag{3}$$

and that of the tail will be

$$C_{DiT} = C_{LT}\left[\varepsilon + \frac{k'C_{LT}}{\pi A_T}\right] \tag{4}$$

Moreover,

$$\varepsilon = \frac{\delta\varepsilon}{\delta\alpha}\frac{C_L}{a} \tag{5}$$

And hence (4) becomes

$$C_{DiT} = \frac{k'C_{LT}{}^2}{\pi A_T} + \frac{\delta\varepsilon}{\delta\alpha} \cdot \frac{1}{a} C_{LT} C_L. \tag{6}$$

The total drag coefficient of the glider will be:

$$C_D = C_{DOW} + C_{DiW} + C_{DF} + (C_{DOT} + C_{DiT})\frac{S_T}{S} \tag{7}$$

Equations (3) and (6) may therefore be substituted in (7), also introducing (1) and (2). The expression for C_D then becomes of the form:

$$C_D = P + QC_L + RC_L^2 \tag{8}$$

where

$$P = C_{DOW} + C_{DF} + C_{DOT}\bar{V}'\frac{\bar{c}}{l_T} + \frac{k}{\pi A}C_{MO}^2\left(\frac{\bar{c}}{l_T}\right)^2 + \frac{k'}{\pi A_T}C_{MO}^2\left(\frac{\bar{c}}{l_T}\right)\frac{1}{\bar{V}'}, \tag{9}$$

$$Q = C_{MO}\left(\frac{\bar{c}}{l_T}\right)\left\{\left[\frac{\delta\varepsilon}{\delta\alpha}\cdot\frac{1}{a} - \frac{2k}{\pi A}\right] + 2\left[\frac{k}{\pi A}\bar{V}'\frac{\bar{c}}{l_T} + \frac{k'}{\pi A_T}\right]\left[\frac{a_1}{a}\left(1 - \frac{\delta\varepsilon}{\delta\alpha}\right) - \frac{H_n}{\bar{V}'}\right]\right\} \quad (10)$$

$$R = \frac{k}{\pi A}\left\{1 - \bar{V}'\frac{\bar{c}}{l_T}\left[\frac{a_1}{a}\left(1 - \frac{\delta\varepsilon}{\delta\alpha}\right) - \frac{H_n}{\bar{V}'}\right]\right\}^2 + \frac{k'}{\pi A_T}\bar{V}'\frac{\bar{c}}{l_T}\left[\frac{a_1}{a}\left(1 - \frac{\delta\varepsilon}{\delta\alpha}\right) - \frac{H_n}{\bar{V}'}\right]^2 + \frac{\delta\varepsilon}{\delta\alpha}\frac{\bar{V}'}{a}\frac{\bar{c}}{l_T}\left[\frac{a_1}{a}\left(1 - \frac{\delta\varepsilon}{\delta\alpha}\right) - \frac{H_n}{\bar{V}'}\right] \quad (11)$$

It should also be noted that a, the lift-curve slope of the complete machine, is strictly also a function of $\bar{V}'$, since

$$a = a_0\left[1 + \frac{S_T}{S}\frac{a_1}{a_0}\left(1 - \frac{\delta\varepsilon}{\delta\alpha}\right)\right] = a_0\left[1 + \bar{V}'\frac{\bar{c}}{l_T}\frac{a_1}{a_0}\left(1 - \frac{\delta\varepsilon}{\delta\alpha}\right)\right] \quad (12)$$

If all other quantities are fixed, P, Q and R are therefore rather complicated functions of $\bar{V}'$.

By the usual analysis:

$$\left(\frac{L}{D}\right)_{max} = \frac{1}{2\sqrt{PR} + Q} \quad (13)$$

$$\text{At } C_L = \sqrt{P/R} \quad (14)$$

and at n times the minimum drag speed

$$\frac{L}{D} = \frac{n^2}{(n^4 + 1)\sqrt{PR} + n^2 Q} \quad (15)$$

Given the quantities in the expressions for P, Q and R other than $\bar{V}'$, and choosing a suitable range of values of the latter, the values given by equations (13) and (15) can be obtained.

5. Calculations

The following values, which might apply to a Standard-Class glider, were assumed:

C_{DOW}	wing profile drag coefficient	0.0076
C_{DOT}	tail profile drag coefficient based on tail area, S_T	0.008
C_{DF}	fuselage drag coefficient based on wing area, S	0.006
k	wing induced drag factor	1.08
k′	tail induced drag factor	1.15
A	wing aspect ratio	15
A_T	tail aspect ratio	5
c / l_T	(wing m.a.c.) ÷ (tail moment arm).	0.2
C_{MO}	moment coeff., aircraft-less-tail . . .	—0.116
a_0	aircraft-less-tail lift curve slope per radian	5.62
a_1	tail lift curve slope per radian . . .	3.38
$\frac{\delta\varepsilon}{\delta\alpha}$	rate of change of downwash at tail with incidence	0.2
H_n	stick-fixed C.G. margin	0.1 and zero
$\bar{V}'$	tail volume coefficient	$\frac{S_T\, l_T}{S\, \bar{c}}$

Inserting these values, L/D was obtained as a function of $\bar{V}'$ for $H_n = 0.1$ and $H_n = 0$, at minimum drag speed and 1.3 times minimum drag speed.

$\bar{V}'$	$H_n = 0.1$ $(L/D)_{max}$	$H_n = 0.1$ (L/D), n = 1.3	$H_n = 0$ $(L/D)_{max}$	$H_n = 0$ (L/D), n = 1.3
0.3	27.40	24.01	28.51	24.88
0.4	27.55	24.11	27.72	24.19
0.5	27.53	24.09	27.58	24.08
0.6	27.47	24.02	27.42	23.93
0.7	27.29	23.86	27.18	23.73

6. Discussion

It has been shown (Ref. 2) that the performance potential of a glider for cross-country flying depends very largely on the maximum lift/drag ratio. However, it is also influenced by the shape of the polar curve and it seemed useful to calculate lift/drag ratios at a typical inter-thermal speed (taken as 1.3 × minimum drag speed) to assess the effect of tail size under these conditions.

Calculations were also made assuming (a) a fairly large stick-fixed C.G. margin and (b) zero stick-fixed C.G. margin, which represents the extreme case permitted by British Civil Airworthiness Requirements. (A rigid glider is assumed, and no distinction is made between C.G. margin and static margin).

Due to the lengthy arithmetic involved, only four cases have been worked out, and it would therefore be imprudent to attempt to generalize too much on this restricted basis. The variations in performance are so small that changes in the glider characteristics (e.g. tail plane aspect ratio) might alter the picture considerably.

The analysis obviously implies various simplifications. It assumes that the only geometrical variable is tail size, the fuselage, for example, being kept the same in all cases. In practice a larger tail, and hence more aft C.G. positions, would enable the length of the fuselage forward of the wing to be decreased with a resulting reduction in skin-friction drag. This would tend to shift the optimum tail volume coefficient to a larger value.

Subject to these considerations, the curves show:

(i) For $H_n = 0.1$, there is a weak maximum at about $\bar{V}' = 0.45$

(ii) If the tail volume coefficient is increased from 0.45 to 0.7, the loss in max. L/D is about 1 %.

(iii) For $H_n = 0$, the smallest tail volumes considered give the best performance.

(iv) For small values of $\bar{V}'$, the performance with $H_n = 0$ is significantly better than with $H_n = 0.1$.

(v) At larger values of $\bar{V}'$, the change in H_n has very little influence on the performance.

(vi) As predicted on qualitative grounds, the performance is not very sensitive to tail volume ratio.

Since gliders are now so refined that designers must seek small gains, it would seem logical to suggest that for competition purposes gliders should have small tailplanes and zero stick-fixed stability. This implies that for a given type the machine would require ballast so as to operate at a fixed C.G. position with different pilot weights. It should be remembered that all the above considerations take no account of the dynamic longitudinal characteristics of the glider. In a powered aircraft, it is known that pilots' subjective impressions of its pleasantness to fly depend not only on the static stability parameters but are also greatly influenced by the characteristics of the short-period oscillation (Ref. 3). No doubt something similar is true for gliders although, as Morelli has observed, the oscillatory behaviour of gliders differs appreciably from that of conventional aeroplanes.

It is clear that a more detailed investigation of the influence of these dynamic effects, taking into account the behaviour of the elevator circuit, will be required to obtain the most satisfactory combination of performance and handling qualities for a particular glider.

References

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