On Optimization of Glider Cross Country Flight Using Thermals

by Dipl.-Ing. O. Niehuss

Presented at the 9th OSTIV Congress, February 1963, Junin (Argentine)

1. Abstract

The purpose of this paper is to determine the optimum gliding speed during flight from thermal to thermal. Optimum gliding speed is defined as the speed resulting in the shortest flight time between two thermals (or two locations, A and B) including the time of climb in the thermals.

It is shown in the results that the optimum gliding speed on the straight course after leaving the thermal is mainly a function of the experienced climb rate in the thermal, the performance characteristics of the glider, and the prevalent wind conditions.

For pilot's convience, regarding cockpit conditions, the optimum gliding speed is expressed in terms of optimum sink rate. Thus, a good rule of thumb could be derived: Under good thermal conditions and negligible wind the optimum sink rate outside the thermal is almost half as great as the experienced climb rate during thermal flight. Flying at the optimum sink rate is equivalent with flying at optimum airspeed.

This rule of thumb is especially valid for high performance gliders and thermal climb rates in excess of 500 ft/min or 2.5 m/sec.

2. Introduction

One of the most challenging goals of a sailplane pilot is to fly long distances. To insure success in his undertaking he must acquaint himself with all the pertinent factors which influence and determine the length of distance flight.

First: There is the time available. Useful time for distance flight is generally limited to the day period during which the sun produces thermals. Therefore, the fastest ship will take the glider pilot over the greatest distance, given a certain useful time period and thermal conditions. Because circling in the thermals appreciably reduces ground speed, there will be an optimum procedure for the pilot to select time spent in the thermals and time and speed of the straight course segments. Application of this optimum procedure, which is affected by the weather conditions, glider performance, etc., will result in the longest distance flight possible at that day.

Second: There is the wind to be considered. The pilot will select a course taking maximum advantage of the prevalent wind in the altitude regime he chooses to fly.

Third: He will ensure himself that he and the ship are properly equipped for the selected course and intended operation for safety sake.

Fourth: The pilote will select the most favourable weather conditions within the season offering the greatest advantages for his distance flight.

While this paper only analyzes the effect of time on a successful distance flight one should not be distracted from the importance and implications of the other, above mentioned factors.

3. Discussion of Problem

Consider the fact, that a glider pilot tries to cover the distance, d, between A and B, representing only a segment of the total distance, within the shortest possible time (see Fig. 1).

$v_{t_{min}}$ = straight course speed resulting in minimum flight time from A to B

$v_{max.\,glide}$ = speed at max. glide

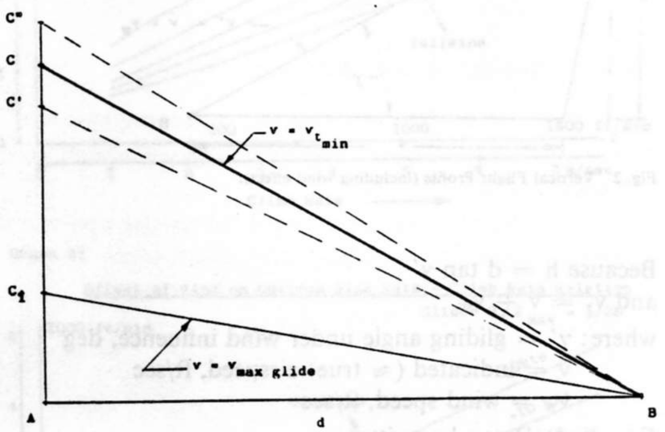


Fig. 1 Vertical Flight Profile (including climb from A to C, or CI, C′, and C″, and straight descending flight from C to B).

Suppose he finds a thermal at A, which he utilizes to climb. He can interrupt the climb at C_I, which is the minimum altitude gain required to reach B at maximum glide. Obviously, he can also reach B from any altitude level (C′, C, C″) above C_I. The problem is to determine the altitude, C above C_I, which results in minimum flight time from A to B. Climbing to an altitude level higher than C_I, the pilot can afford to fly faster on the straight course segment, because of the additional altitude gain above that required for maximum glide. However, climbing to higher altitude requires time, which, to a certain degree, offsets the advantage gained by the greater speed possible because of the additional altitude gain.

It will be shown, that there is one and only one altitude or instant, where the pilot should interrupt the climb to arrive at B in the shortest possible time, climb time + segment time.

To establish a procedure, with which the glider pilot can optimize this one flight segment with respect to flight time under cockpit conditions is the purpose of this paper. A succession of many of these optimized flight legs results in an optimized distance flight under given atmospheric conditions.

4. Analysis

To simplify the calculations several assumptions were necessary:

1) constant climb rate during thermal flight
2) constant true airspeed during straight decending flight from thermal to thermal (here A and B)
3) no downdrafts on the straight course
4) constant wind direction and velocity during flight
5) constant air density
6) the drag coefficient c_D is defined by a constant, K_I, and a term defined as $K_2 c_L^2$, where K_2 is a function of effective wing aspect ratio and shape.

$$c_D = K_I + K_2 c_L^2$$

The main reasons for the above mentioned assumptions are:

1) unsurmountable difficulties to account for the unsteady and irregular air flow patterns within the thermal
2) to provide pilots in the air with a simple method to approximate best indicated airspeed from a graphical display which can be easily interpreted and utilized under cockpit conditions.

The total time, t_{tot}, spent during flight from A to B, considering the above mentioned assumptions, is (see Fig.2 below)

$$t_{tot} = \frac{h}{v_c} + \frac{d}{v'} \quad \text{(sec)} \qquad (1)$$

where h = altitude gain, ft
v_c = climb rate, ft/sec
d = distance, ft
v' = ground speed corrected for wind, ft/sec

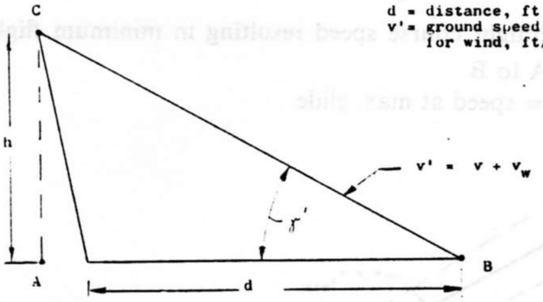


Fig. 2 Vertical Flight Profile (including wind effect).

Because $h = d \tan \gamma'$
and $v' = v + v_w$
where: γ' = gliding angle under wind influence, deg
v = indicated ($\approx$ true) airspeed, ft/sec
v_w = wind speed, ft/sec

Equation (1) can be written as

$$t_{tot} = d\left[\frac{\tan \gamma'}{v_c} + \frac{1}{v + v_w}\right] \quad \text{(sec)} \qquad (2)$$

Only head or tail wind is considered in Equation (2)

Because $\tan \gamma' = \frac{v}{v + v_w} \cdot \varepsilon$

and $\varepsilon = \frac{K_1 + K_2 c_L^2}{c_L}$

and $v = \sqrt{\frac{2W}{\rho S}} \frac{1}{\sqrt{c_L}} = \frac{K}{\sqrt{c_L}}$

where ε = gliding angle, deg
K_1 = minimum drag coefficient, c_{D_0}
$K_2 = \frac{1}{\pi A e}$ (e = wing efficiency)
A = wing aspect ratio
c_L = lift coefficient
ρ = air density, slugs/ft^3
W = airplane weight, lbs
S = wing area, ft^2
K = factor $= \sqrt{\frac{2W}{\rho S}}$, ft/sec

Equation (2) can be modified to read

$$t_{tot} = \frac{d}{K/\sqrt{c_L} + v_w} \cdot \left[\frac{K/\sqrt{c_L} \cdot \frac{K_1 + K_2 c_L^2}{c_L}}{v_c} + 1\right] \quad \text{(sec)} \qquad (3)$$

Dividing Equation (3) by d yields an expression of the reciprocal value of the ground speed. Therefore, the problem and its solution becomes independent of the distance d. Optimizing the ground speed results in a procedure applicable to any given leg distance or climb altitude the glider pilot may choose or reach through thermal climb.
We have

$$\frac{1}{v_{ground}} = \frac{t_{tot}}{d} = \frac{1}{K/\sqrt{c_L} + v_w}\left[\frac{K/\sqrt{c_L} \cdot \frac{K_1 + K_2 c_L^2}{c_L}}{v_c} + 1\right] \quad \left(\frac{\text{sec}}{\text{ft}}\right) \qquad (4)$$

Differentiating Equation (4) with respect to c_L yields after rearranging and equating to zero:

$$\frac{d\left(\frac{t_{tot}}{d}\right)}{dc_L} = 0 = \frac{2 c_L^{3/2} K_2 c_L}{(K/\sqrt{c_L} + v_w) c_L^3} - \frac{(K_1 + K_2 c_L^2)\, 3/2\sqrt{c_L}}{(K/\sqrt{c_L} + v_w) c_L^3}$$
$$+ \frac{(K_1 + K_2 c_L^2) K}{c_L^3 \, 2(K/\sqrt{c_L} + v_w)^2} + \frac{v_c}{c_L^{3/2}(K/\sqrt{c_L} + v_w)^2} \qquad (5)$$

Because the second derivative is positive (necessary condition), Equation (5) relates the desired optimum lift coefficient, c_L, to the experienced climb rate, v_c, in the thermal. The wind speed, v_w, is a parameter. (Only head- or tail wind is considered).

Because c_L determines airspeed and descent rate, v_s, one can express the optimum airspeed or descent rate, whatever one chooses, as a function of the climb rate, v_c. For reasons, which will become evident later, it is advantageous to calculate the optimum sink rate or descent rate of the straight flight path as a function of the climb rate in the thermal.

Solving Equation (5) for v_c yields

$$v_c = (K/\sqrt{c_L} + v_w)\frac{K_1 + K_2 c_L^2}{c_L}\left[3 - \frac{4 K_2 c_L^2}{K_1 + K_2 c_L^2} - \frac{K/\sqrt{c_L}}{K/\sqrt{c_L} + v_w}\right] \qquad (6)$$

Under "no wind" conditions, Equation (6) becomes

$$v_w = 0: \quad v_c = 2K\left[\frac{K_1 + K_2 c_L^2}{c_L^{3/2}} - 2 K_2\sqrt{c_L}\right] \qquad (7)$$

A useful procedure to evaluate Equation (6) and (7) is to compute v_c as a function of c_L. The lift coefficient then determines the corresponding optimum airspeed or descent rate of a given glider and altitude level. To perform the computations, one has to insert into Equations (6) and (7) the aerodynamic coefficients, K_1 and K_2, of the sailplane and the value of K, which is determined by the air density, ρ, (ρ can be an average for the used altitude regime), sailplane gross weight, W, and wing area, S. By varying v_w, one can account for different wind velocities (head- or tail wind only)

Because v_c and v_s are linear functions of K ($= \sqrt{\frac{2W}{\rho S}}$) at least for the cases where the wind velocity is small compared to the glider speed, changes of glider gross weight, W, and of flight altitude or air density, ρ, have no (under „no wind" conditions) or a small effect on the v_c — v_s relationship.

Therefore, the graphs can be utilized by different pilots flying the same ship with different gross weights, at different altitudes. In all cases where the horizontal wind is small compared to the glider speed, which becomes greater than the speed of maximum glide under good thermal conditions (v_c > 500 ft/min), errors introduced by the non-linearity should be disregarded.

5. Results

Graph 1 shows the optimum sink rate, v_s, as a function of climb rate, v_c, during thermal flight under „no wind" conditions. The three plots represent three different sailplane configurations, which are identified by the following data:

Airplane weight: 650 lbs,
Wing area: 161 ft^2

A) max. glide angle = 1/36,
$K_1 = 0.012$, $K_2 = 0.0159$, aspect ratio = 20

B) max. glide angle = 1/28,
$K_1 = 0.015$, $K_2 = 0.0212$, aspect ratio = 15

C) max. glide angle = 1/21,
$K_1 = 0.018$, $K_2 = 0.0318$, aspect ratio = 10

Because gross weight and air density have a negligible effect on the results, arbitrary but representative values for weight and density have been chosen. All three sailplanes have a weight of 650 lbs, a wing area of 161 ft^2, and aspect ratios of 20, 15, and 10 respectively. A flight level of 4000 ft ($\rho = 0.002112$ slugs/ft^3) was selected.

Graphs 2, 3, and 4 show the effect of wind on the optimum sink rate of all three gliders, A, B, and C.

6. Discussion of the Graphs

Graph 1, v_c — v_s relation under „no wind" condition, shows the effect of aerodynamic cleanness on the v_c — v_s relation. Note, that for thermal conditions favourable to distance flights with thermals between 600 and 1200 ft/min, the computed optimum sink rate during flight on the straight course is about half as great as the climb rate in the thermal.

Because
$$v_c = 2K\left(\frac{\varepsilon}{\sqrt{c_L}} - 2K_2\sqrt{c_L}\right)$$

and
$$v_s = \frac{K}{\sqrt{c_L}\,(1+\varepsilon/2)}\ \frac{\varepsilon}{(1+\varepsilon)}$$

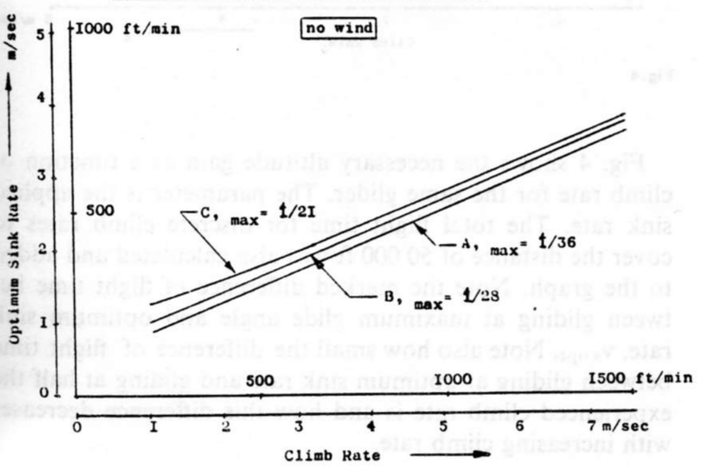
dividing v_s into v_c yields
$$\frac{v_c}{v_s} = \frac{2(\varepsilon - 2K_2c_L)\,(1+\varepsilon/2)\,(1+\varepsilon)}{\varepsilon}$$

Because ε and $\varepsilon/2$ are small compared to 1 and because $2K_2c_L$ is small compared to ε in the lower c_L range (high speed), one can see that

$$\frac{v_c}{v_s} = 2/1 \quad \text{or} \quad \frac{dv_c}{dv_s} = 2$$

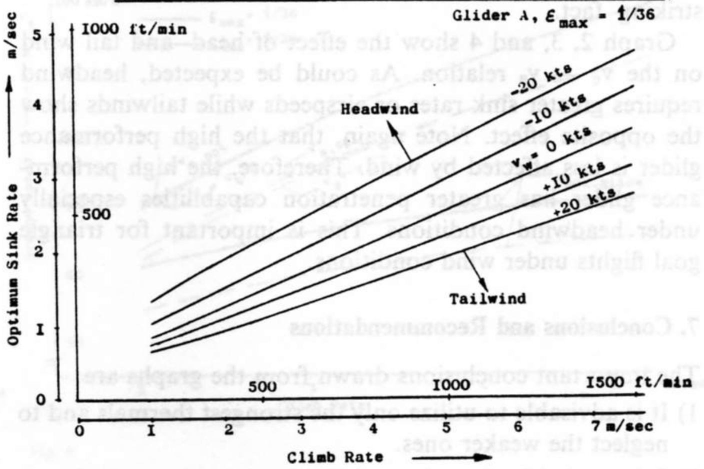
Graph 1:

Optimum Sink Rate - Climb Rate Relation



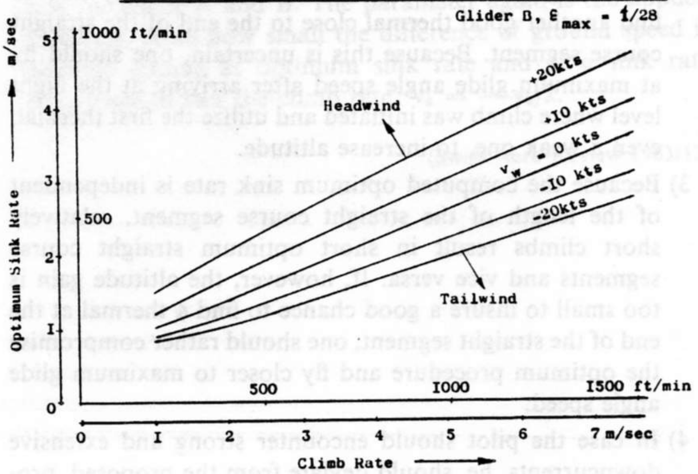
Graph 2:

Effect of Wind on Optimum Sink Rate - Climb Rate Relation, Glider A, ε_{max} 1/36



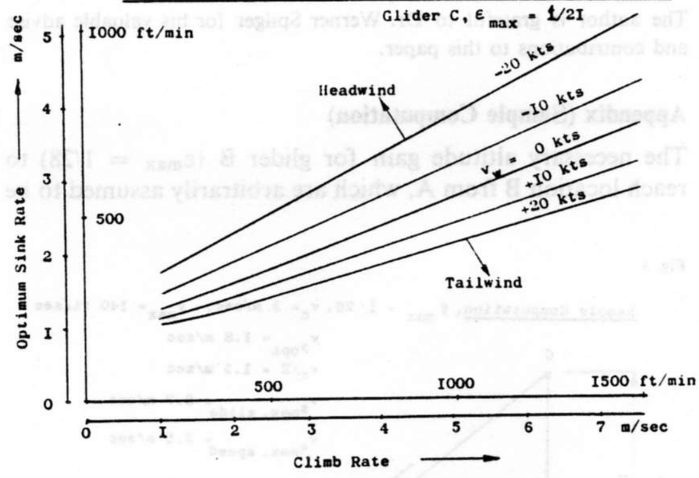
Graph 3:

Effect of Wind on Optimum Sink Rate - Climb Rate Relation Glider B, ε_{max} = 1/28



Graph 4:

Effect of Wind on Optimum Sink Rate - Climb Rate Relation Glider C ε_{max} = 1/21



This is true especially for the high performance glider, (low ε values at high speed).

It is also evident, that the greater the climb rate experienced in the thermal, the greater the sink rate (or the greater the forward speed) that one can afford to fly.

In the region of higher climb rates than 500 ft/min the effect of aerodynamic cleanness on the v_c — v_s values become less and less pronounced. However, because the high performance gliders associate a much greater airspeed with a given sink rate, their superiority for distance flight purposes over the lower performance gliders becomes a striking fact.

Graph 2, 3, and 4 show the effect of head- and tail wind on the v_c — v_s relation. As could be expected, headwind requires greater sink rates or airspeeds while tailwinds show the opposite effect. Note again, that the high performance glider is less affected by wind. Therefore, the high performance glider has greater penetration capabilities especially under headwind conditions. This is important for triangle goal flights under wind conditions.

7. Conclusions and Recommendations

The important conclusions drawn from the graphs are:

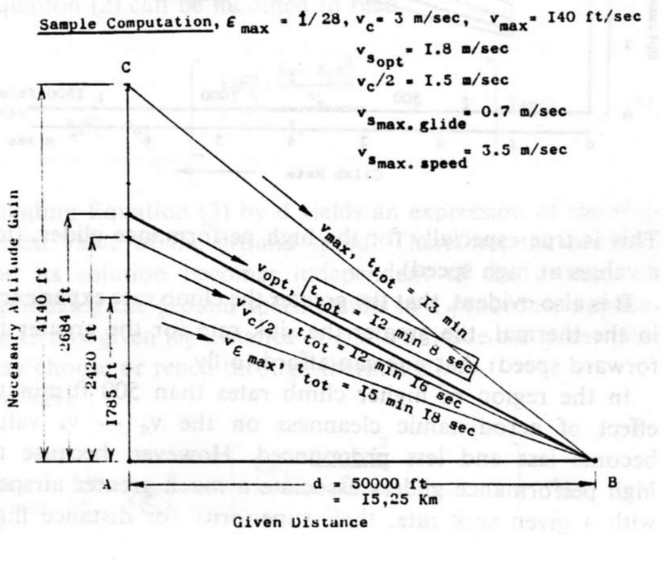
1) It is advisable to utilize only the strongest thermals and to neglect the weaker ones.
2) Select an altitude regime of flight, where the updrafts are strongest and do not continue to climb when the rate is decreasing, provided that one can be reasonably sure to find another good thermal close to the end of the straight course segment. Because this is uncertain, one should fly at maximum glide angle speed after arriving at the flight level where climb was initiated and utilize the first thermal, even a weak one, to increase altitude.
3) Because the computed optimum sink rate is independent of the length of the straight course segment, relatively short climbs result in short optimum straight course segments and vice versa. If, however, the altitude gain is too small to insure a good chance to find a thermal at the end of the straight segment, one should rather compromise the optimum procedure and fly closer to maximum glide angle speed.
4) In case the pilot should encounter strong and extensive downcurrents, he should deviate from the proposed procedure during that period and fly at maximum glide angle speed under those conditions.

The author is grateful to Dr. Werner Spilger for his valuable advise and contributions to this paper.

Appendix (Sample Computation)

The necessary altitude gain for glider B ($\varepsilon_{max} = 1/28$) to reach location B from A, which are arbitrarily assumed to be

Fig. 3



50 000 ft (or 15.25 Km) apart, has been calculated as well as the total time elapsed, under the assumption that the average climb rate, v_c, during thermal flight is 3 m/sec, and that the glider pilot uses various (but constant average in each case) descent rates, v_s, during straight flight from C to B.

If the pilot chooses to fly from A to B using max. glide speed, v_{max} glide, the altitude gain necessary to reach B is 1785 ft and the total time elapsed is 15 min 18 sec.

If he wants to fly at maximum allowable cruising speed. e.g., (which is assumed here to be 140 ft/sec) he has to gain 4140 ft. The total time elapsed would be 13 min 0 sec.

Flying at optimum glide speed requires to fly at a sink rate of 1.8 m/sec and to gain 2684 ft altitude. The speed during straight flight would be 110 ft/sec (121 km/h or 75 mph) and the total time only 12 min 8 sec.

If the pilot chooses to apply the „rule of thumb" mentioned in the abstract of this paper, he would fly at a sink rate equivalent to half the climb rate, $v_s = 1.5$ m/sec; his speed on the straight course would be 102 ft/sec (or 111 Km/h, 70 mph), the altitude gain required 2420 ft and the total time elapsed 12 min 16 sec, which is only 8 seconds longer than flying at optimum gliding speed. The time increment is in the order of 1%. The magnitude of this error compared to the magnitude of other errors, like instrument reading errors, etc., leads one to the conclusion that applying the optimum flight procedure requiring the preparation and reading of charts etc., may not be warranted because the „rule of thumb" yields such good results and can be applied without much difficulties by glider pilots who are not particularly on firm ground in mathematics.

Under wind conditions, the error introduced by this rule would magnify but still would yield better results than flying at best gliding angle, or maximum speed.

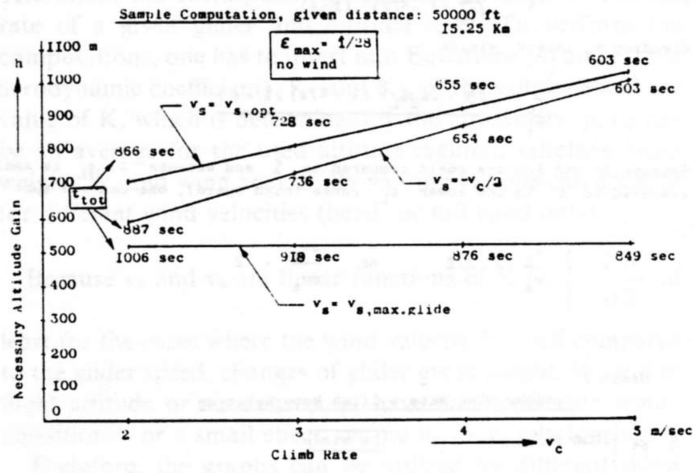


Fig.4

Fig. 4 shows the necessary altitude gain as a function of climb rate for the same glider. The parameter is the applied sink rate. The total flight time for discrete climb rates to cover the distance of 50 000 ft was also calculated and added to the graph. Note the marked difference of flight time between gliding at maximum glide angle and optimum sink rate, $v_{s\,opt}$. Note also how small the difference of flight time between gliding at optimum sink rate and gliding at half the experienced climb rate is and how this difference decreases with increasing climb rate.

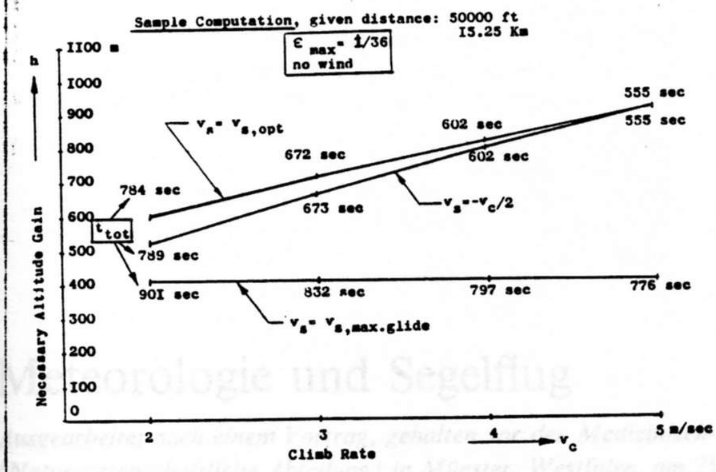


Fig. 5

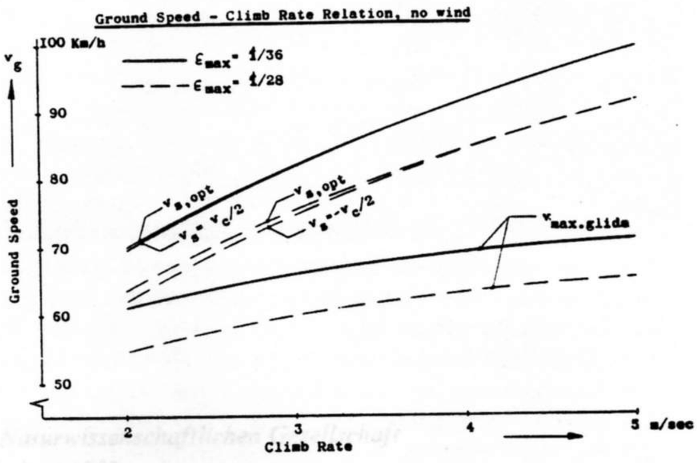


Fig. 6

Fig. 5 shows the results of the computations of the high performance glider A ($\varepsilon_{max} = 1/36$).

Fig. 6 shows the ground speed, v_g, as a function of climb rate for glider A and B. The parameter again is the applied sink rate. Note how small the difference of ground speed is between gliding at optimum sink rate and at a sink rate equivalent to half the climb rate: $v_s = -v_c/2$.

(Swiss Aero Review 1963/12)