Gust alleviation factors for sailplanes

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Summary

In this report an analysis is made of the gust alleviation factors given in existing airworthiness requirements for sailplanes. It is concluded that the alleviation factor formula contained in the OSTIV requirements should be preferred over the simple relations given in British and U.S. requirements. A few general remarks are made with regard to the influence of altitude and finite structural stiffness on gust-imposed loads.

List of symbols

$\bar{c}$	=	mean wing chord	(m)
g	=	acceleration of gravity	(m/sec²)
Δn	=	incremental load factor	
s	=	dimensionless time; $s = \frac{Vt}{\bar{c}}$	
t	=	time	(sec)
u	=	gust velocity	(m/sec)
w	=	airplane vertical velocity	(m/sec)
F	=	gust alleviation factor (defined in eq. (1))	
H	=	gust gradient-distance (expressed in wing chords); for the definition of H see fig. 2	
S	=	wing area	(m²)
U	=	maximum gust velocity	(m/sec)
V	=	flying speed	(m/sec)
W	=	airplane weight	(kg)
W/S	=	wing loading	(kg/m²)
μg	=	mass ratio parameter, $\dfrac{W/S}{\frac{1}{2}\rho g \frac{dC_L}{d\alpha} \cdot \bar{c}}$	
φg, φw	=	dynamic lift functions	
ρ	=	air density	(kgsec²m-⁴)

1. Introduction

The present OSTIV Airworthiness Requirements for Sailplanes contain gust loading cases which have been adopted from the Polish requirements.

These gust loading cases are essentially different from those given in other requirements, for example the British and U. S. regulations.

In the case of high wing loads, the OSTIV requirements may lead to higher design gust loads. This effect, which is of course disliked by sailplane designers, has raised some doubt as to the suitability of the OSTIV gust cases.

A discussion on the suitability of a certain gust requirement or a comparison between different requirements, is complicated due to two facts. In the first place, our physical knowledge of the phenomenon of atmospheric turbulence is very limited.

In the second place, existing gust requirements are given in such a form that the theoretical basis underlying them can hardly be recognized.

In this paper a survey will be given of the basic assumptions made in the derivation of some existing gust requirements for sailplanes. A qualitative comparison of these assumptions may give information about the suitability of the different requirements.

Comparisons will be restricted to the symmetrical gust loading cases as given in the OSTIV, British and U. S. requirements for sailplanes. It is thought that requirements from other origin do not contain elements which are essentially different.

In all requirements mentioned, the incremental load factor due to gusts may be calculated from[1]

$$\Delta n = \frac{\frac{1}{2}\rho \frac{dC_L}{d\alpha} \cdot U.V}{W/S} \cdot F \qquad (1)$$

in which U is the vertical gust velocity, V the flying speed at the gust encounter and F is the so-called "gust alleviation factor".

The different requirements specify different values for U and V. In fact, the British requirements specify a strong gust occurring at low speed, the U. S. requirements specify a relatively weak gust occurring at high speed, whereas the OSTIV requirements contain both a "strong gust-low speed"—and a "weak gust-high speed"—case.

Of course these differences in definition reflect an important difference in philosophy, which may be very interesting to discuss.

However, it may easily be shown that for the present purpose the most important difference, resulting in different relations between load factor and wing loading, stems from the different expressions given for the gust alleviation factor F.

In the next chapter some information will be given on the backgrund of existing alleviation factors.

2. The background of existing "gust alleviation factors"

When an aircraft flying at speed V enters a sharp-edge gust (see fig. 1) with gust velocity U, the angle of incidence will be increased by an amount $\Delta\alpha = U/V$. Neglecting aerodynamic inertia it may easily be found that this increase of angle of incidence will lead to an instantaneous increase of the load factor by:

$$\Delta n = \frac{\frac{1}{2}\rho \cdot \frac{dC_L}{d\alpha} \cdot U.V}{W/S} \qquad (2)$$

In the earliest stages of gust research in the United States the above "sharp-edge gust formula" was used to reduce gust loads, measured in flight, to "effective gust velocities".

The word "effective" was used to indicate the fictitious nature of the assumed sharp-edge profile.

The effective gust velocities found in this way were used to predict gust loads for new aircraft.

[1] According to OSTIV, the factor Δn has to be multiplied by 1.2 in order to take pitching motion influence into account.

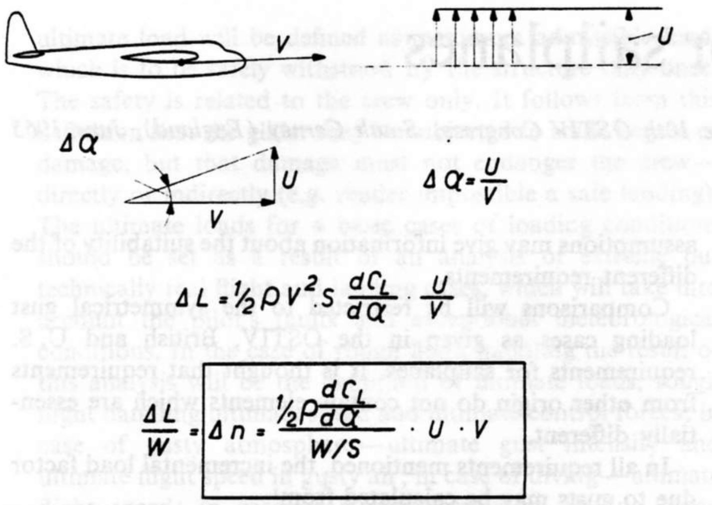

Fig. 1.—The sharp-edge gust formula.

This simple procedure gave reasonable results as long as the dimensions and basic parameters of the aircraft to which it was applied did not differ too much.

However, around 1935 a new class of aircraft with much larger dimensions and higher wing loadings was introduced (civil transport and bombers). When at the sametime it was decided to establish structural requirements for sailplanes it became clear that the simple procedure outlined above insufficiently accounted for the different response characteristics of different aircraft to be useful for such a broad spectrum of airplane types.

Both the finite gradient-distance of a "real" gust and the aerodynamic inertia in changing the lift distribution have the effect that the aircraft "has the time" to respond by an upward movement before the maximum gust-load is reached, thus reducing the load induced by the gust.

In order to evaluate this effect several calculations have been carried out to establish the response of an aircraft to discrete gusts with different profiles and different gradient distances.

Although some investigators have included the influence of pitch-response and/or finite structural flexibility in their calculations, this paper will deal only with the calculated responses of an aircraft having one degree of freedom, viz. vertical translation.

The Appendix gives some details of these calculations.

With regard to the maximum incremental load factor, Δn_{max}, caused by a gust, it was found that Δn_{max} could be expressed as:

$$\Delta n_{max} = \frac{\frac{1}{2}\rho \frac{dC_L}{d\alpha} U.V}{W/S} \cdot F(\mu_g, H) \; . \qquad (3)$$

In other words, the response-freedom of the aircraft leads to an alleviation of the load induced by a gust, compared to that calculated from the "sharp-edge gust formula" (1), by a factor F (μ_g, H). The factor F, which will be called the "gust alleviation factor" is a function of the mass ratio parameter μ_g, defining the response characteristics of the aircraft, and the gradient distance H, defining the gradient length of the assumed gust.[2]

[2] The dependence of F upon Aspect Ratio will be neglected here, see the Appendix.

The calculated values for F, which in most cases cannot be given in an analytical form, depend on the shape of the assumed gust profile (see fig. 2).

To define a gust loading case for Airworthiness Requirements, it is in principle sufficient to establish a design gust of a certain strength, shape and length; the calculation of the load induced by this gust on a particular aircraft may be left to the designer. Most existing Requirements, however, establish a maximum gust velocity U and a certain relation for the gust alleviation factor F; the induced loads may be calculated from eq. (3).

In fact, this procedure may be seen as a "modified sharp-edge gust procedure": the loads calculated by the "sharp-edge gust formula" (2) are multiplied by a factor F, which takes the influence of aircraft response into account.

In the following a survey will be given of the way in which existing alleviation factors for sailplanes were derived from calculated responses to discrete gusts.

According to ref. 1 and ref. 2 the derivation of the alleviation factor contained in the U. S. requirements for sailplanes may be summarized as follows. Calculations were made of the response of an aircraft to a ramp-type gust. Both the influences of finite gradient distance and aerodynamic inertia were included. To simplify calculations the assumption was made that the maximum load factor occurs before or at the moment that the maximum gust velocity is reached. Because of this assumption the calculated maximum load is independent of the way in which the gust velocity varies after reaching its maximum value; thus the calculated alleviation factor F (μ_g, H) is both valid for a flat-topped gust (fig. 2a) and a triangular gust (fig. 2c).

According to the American philosophy, which will be discussed in the next chapter, the gradient distance is a constant times the wing chord, or in other words H is

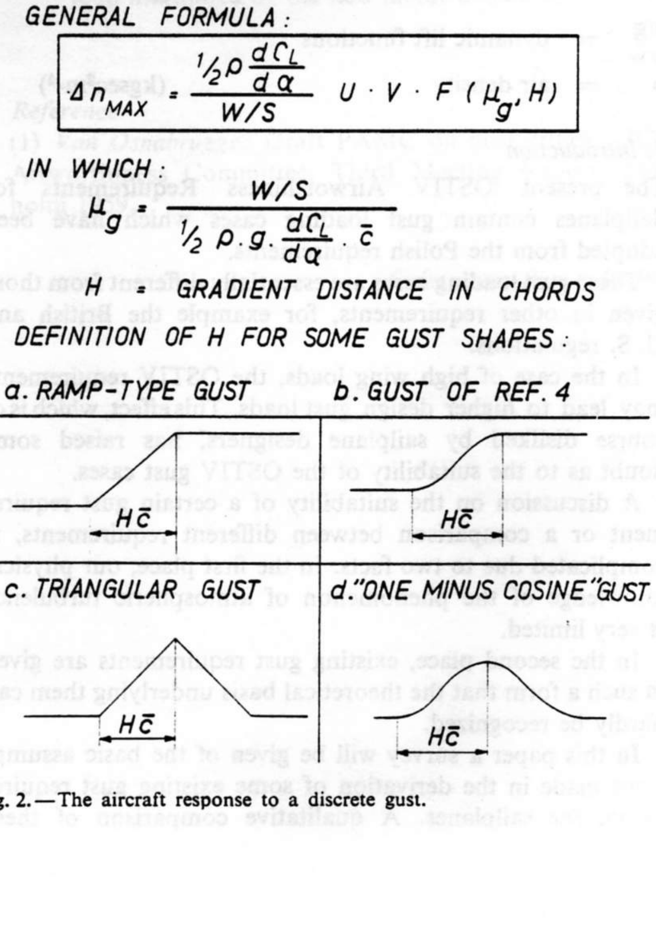

Fig. 2.—The aircraft response to a discrete gust.

constant. The value taken was H = 10. The calculated value of F is, consequently, only a function of the mass ratio parameter μ_g.

The next step was to assume a certain constant relation between mass ratio parameter μ_g and wing loading W/S; in this way the alleviation factor found could be expressed as a function of the "more convenient" parameter W/S instead of μ_g.

Finally, the alleviation curve found was "normalized" to yield a value F = 1 for W/S = 16 p. s. f., which is the wing loading of the Boeing B-247, an aircraft used in gust investigations. This led to the rather strange result that the "alleviation" factor may be larger than one. Of course, the prescribed gust velocity had to be reduced correspondingly, so that the product of alleviation factor and gust velocity was not affected by the normalisation.

The alleviation factor found has the very simple form: $F = 0.5\ (W/S)^{1/4}$, where W/S is expressed in p. s. f. This relation is given in the U. S. requirements. The British requirements for sailplanes (BCAR-E) state that in the absence of better information the gust alleviation factor shall be taken as $F = 0.3\ (W/S)^{1/4}$, where W/S is expressed in p. s. f. Obviously, this relation was taken directly from the U. S. requirement, except for the normalisation procedure, which is not carried out in the British case.

At this stage of the analysis the following remarks should be made.

1. The gust alleviation factors discussed above are based on a ramp-type gust with a gradient distance of ten chords.
2. Although the assumed relation between W/S and μ_g is unknown to the author, the validity in the case of sailplanes, with very low wing loadings and very small wing chords is doubtful. Moreover, for a certain airplane the relation can only hold for one altitude. In other words, the dependence on air density p of μ_g and hence of the alleviation factor F is neglected.

According to the Polish (and consequently OSTIV) philosophy the gradient distance of the gust is proportional to the gust strength:

$$H\ \bar{c}\ (m) = U\ (m/sec).$$

Using this expression the alleviation factor formula given in the OSTIV Requirements may be converted into

$$F = \frac{\mu_g}{H}\left[1 - e^{-\frac{H}{\mu_g}}\right], \quad F \leq 0.6.$$

It may be deduced (see the Appendix) that this factor is based on the calculated response to a gust with either the flat-topped profile of fig. 2a or the triangular gust of fig. 2c. For large values of H/μ_g i. e. large gradient distances, the alleviating effect of aerodynamic inertia is neglected, whereas for values of $\frac{H}{\mu_g} < 1.2$ the combined influence of finite gradient and aerodynamic inertia is approximated to yield a constant alleviation factor F = 0.6.

An impression of the errors introduced by these approximations may be derived from fig. 4.

In this figure the OSTIV approximation is superposed on the "exact" results for a triangular gust, as calculated in R & M No. 2970 by Zbrozek (ref. 3).

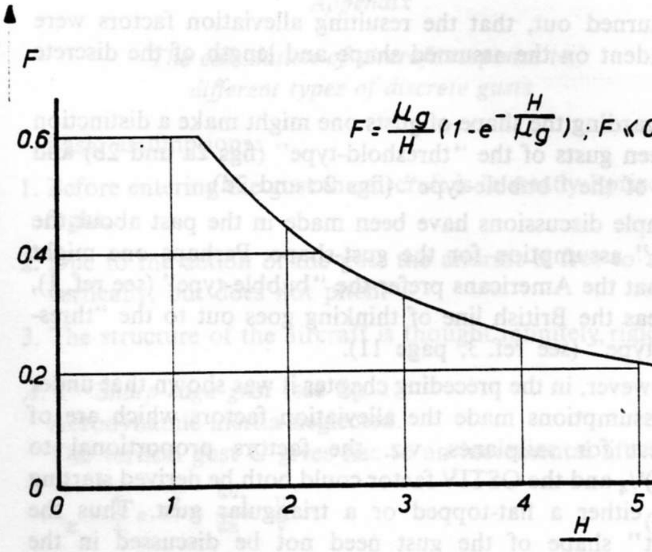

Fig. 3.—The OSTIV alleviation curve.

Keeping in mind that for sailplanes the value of μ_g will probably always be lower than 15, whereas $\mu_g = 8$ is a normal figure, it can be seen that for gradient distances of more than 20 chords, say about 20 metres, the agreement is excellent. For very short gradient distances the agreement is worse, but still quite acceptable.

3. The shape and gradient-distance of discrete gusts

In the preceding chapter the deduction of existing gust-alleviation factors from gust-response-calculations was discussed.

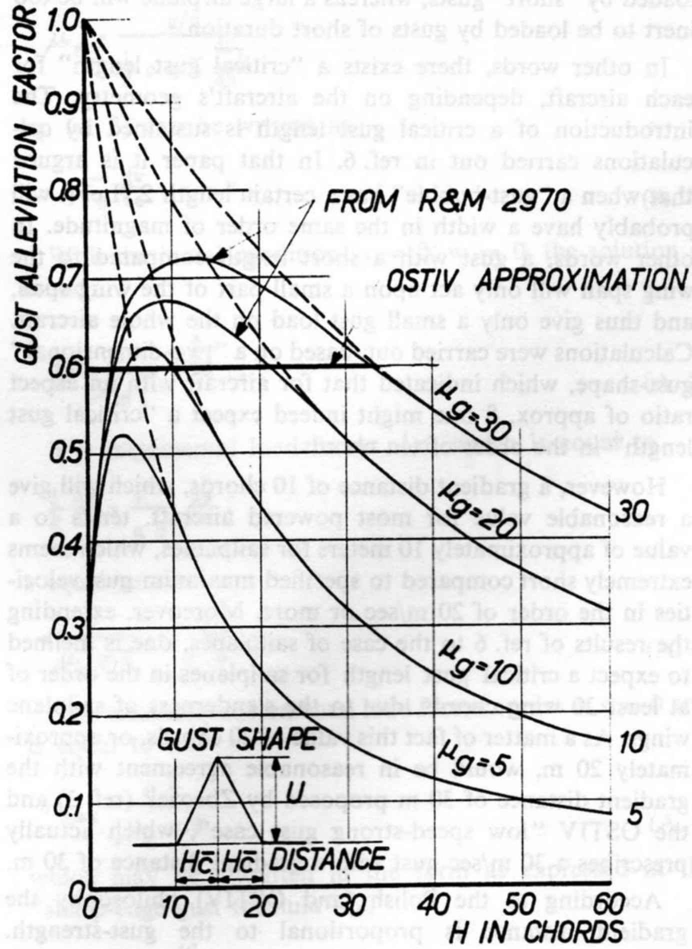

Fig. 4.—Comparison of the OSTIV alleviation factor with results of Zbrozek.

It turned out, that the resulting alleviation factors were dependent on the assumed shape and length of the discrete gust.

Regarding the shape of gusts one might make a distinction between gusts of the "threshold-type" (figs 2a und 2b) and gusts of the "bubble-type" (figs 2c and 2d).

Ample discussions have been made in the past about the "best" assumption for the gust-shape. Perhaps one might say that the Americans prefer the "bubble-type" (see ref. 1), whereas the British line of thinking goes out to the "threshold-type" (see ref. 3, page 11).

However, in the preceding chapter it was shown that under the assumptions made the alleviation factors which are of interest for sailplanes, viz. the factors proportional to $(W/S)^{1/4}$ and the OSTIV factor could both be derived starting from either a flat-topped or a triangular gust. Thus the "right" shape of the gust need not be discussed in the present paper.

In fact, the establishment of the gust-gradient-distance is of much more interest here. It should be kept in mind that the assumption of a short gradient distance leads to an underestimation of the alleviating effect.

As mentioned before, the American philosophy goes out from a gradient distance which is a certain number of times the wing-chord. At first sight one is inclined to believe that this assumption is largely suggested by the consequent simplified result (F is a function of μ_g only). However, one might defend the assumption by posing that a small aircraft can move upward easily with a "long" gust, thus will only be loaded by "short" gusts, whereas a large airplane will be too inert to be loaded by gusts of short duration.

In other words, there exists a "critical gust length" for each aircraft, depending on the aircraft's geometry. The introduction of a critical gust length is sustained by calculations carried out in ref. 6. In that paper it is argued that when a "gust-bubble" has a certain length $2\,H\,\bar{c}$, it will probably have a width in the same order of magnitude. In other words, a gust with a short length compared to the wing span will only act upon a small part of the wing span, and thus give only a small gust load on the whole aircraft. Calculations were carried out, based on a "two-dimensional" gust-shape, which indicated that for aircraft with an aspect ratio of approx. 8 one might indeed expect a "critical gust length" in the order of ten chords.

However, a gradient distance of 10 chords, which will give a reasonable value for most powered aircraft, tends to a value of approximately 10 meters for sailplanes, which seems extremely short compared to specified maximum gust velocities in the order of 20 m/sec or more. Moreover, extending the results of ref. 6 to the case of sailplanes, one is inclined to expect a critical gust length for sailplanes in the order of at least 20 wing chords, due to the slenderness of sailplane wings. As a matter of fact this value of 20 chords, or approximately 20 m, would be in reasonable agreement with the gradient distance of 30 m proposed by Zbrozek (ref. 3) and the OSTIV "low speed-strong gust case", which actually prescribes a 30 m/sec gust with a gradient distance of 30 m.

According to the Polish (and OSTIV) philosophy the gradient distance is proportional to the gust-strength. Although this assumption is physically acceptable, one is inclined to believe that in the OSTIV "high speed-weak gust" case the consequent gradient distance of 4 m is very low, especially seen in the light of the previous discussion.

4. Discussion

In the preceding chapters it has been shown that all gust loading cases prescribed in the requirements investigated are based on the occurrence of a single, discrete gust, the aircraft being considered to respond only in a vertical motion. Differences were found in the gradient distance of the applied gust and in the simplifications made to derive a simple alleviation factor formula from the calculated results.

In the case of the U. S. and British requirements the assumed gust has a gradient distance of 10 chords or approximately 10 meters. This figure seems rather small, both in relation to the aircraft's span and to the strength of gusts in the order of 20 m/sec.

A short gradient distance leads to an underestimation of the alleviating effect.

The assumed constant relation between W/S and μ_g is very dubious. Moreover, it neglects the influence of altitude on μ_g and hence on the alleviation factor.

It was shown that the OSTIV alleviation curve was a very good approximation of the response calculations. The gradient distance was assumed to be proportional to the gust strength, yielding the quite acceptable value of 30 metres or approximately 30 chords in the case of strong gusts.

Hence it must be concluded that the OSTIV alleviation curve should be preferred over the expressions as given in the British and U. S. requirements. The increase of gust loads with wing loading is the consequence of a physical reality, which will have to be accepted.

Having reached this point, it becomes necessary to make some general remarks with regard to the assumptions. which are the same for all requirements mentioned.

In the first place there is the influence of altitude. Although the OSTIV alleviation factor accounts for differences in altitude, the gust cases specified in the requirements relate to sea-level. It can be easily shown that for the same equivalent flying speeds and gust velocities larger gust loads will occur at higher altitudes due to the increase of F with altitude. This may be illustrated by a numerical example.

A sailplane having a wing load of 23 kg/m² has a mass ratio of appr. 7.5 at sea level. Taking a gradient distance of 30 chords and using the OSTIV alleviation curve (fig. 3) a factor $F = 0.25$ is found.

The mass ratio increases with altitude, due to the decrease of the air density. At an altitude of 3000 metres the mass ratio is appr. 10, giving a value of $F = 0.32$.

In other words, the same equivalent gust gives rise to an incremental load which is $\frac{0.32}{0.25}$ times or 28 % larger at an altitude of 3000 metres than at sea level.

The increasing popularity of high altitude flying makes it necessary to consider the introduction of a high-altitude gust case in the airworthiness requirements. In the second place there are the assumptions of infinite structural stiffness and absence of pitch-response motions. Probably the influence of pitch-response will always be very small, at least for sailplanes with a normal configuration.

With regard to flexibility effects, the attention should be drawn to a recent D. V. L. publication by Hacklinger and Pietrass (ref. 7) in which it is shown that flexural responses may lead to a considerable amplification of gust-imposed loads, even for present-day sailplanes. In fact, the increase of flying speeds, the use of very thin and slender wings and the introduction of new structural materials with a large strength-stiffness ratio make it highly probable that these flexural influences may become of increasing importance and can no longer be disregarded.

However, it is the author's opinion that it will hardly be possible to take these effects into account in a satisfactory way by means of a simple modification of the present-day gust approach.

In the last few years new techniques, based on Power-Spectral methods have been developed for the prediction of gust-imposed loads. These methods are based on the physically consistent picture of atmospheric turbulence as a randomly varying, stochastic process. They offer the possibility to take flexibility effects into account by rather simple means. It should be possible to modify these methods, which are used nowadays only for the design of advanced powered aircraft, in such a way that they can be used for the design of sailplanes.

5. Conclusions

1. The gust alleviation factor formula contained in the OSTIV requirements should be preferred over the simple relations expressed as a function of W/S, as given in the U. S. and British requirements for sailplanes.
2. The necessity should be investigated of introducing a high-altitude gust case in the OSTIV airworthiness requirements.
3. Structural flexibility will probably become of increasing importance with regard to gust-imposed sailplane loads. Power-spectral methods may be used to take these influences into account in a realistic way.

6. References

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Appendix

The calculation of aircraft response to different types of discrete gusts

Basic assumptions:

1. Before entering the gust the aircraft is in steady horizontal flight.
2. Due to the action of the gust the aircraft is free to move vertically, but does not pitch.
3. The structure of the aircraft is thought infinitely rigid.

A. 1 Sharp-edge gust (see fig. 1)

Aerodynamic inertia neglected.

The vertical gust U gives rise to an incremental lift force:

$$\Delta L_g = \frac{1}{2}\rho V^2 S \frac{dC_L}{d\alpha}\frac{U}{V} \qquad \text{(A. 1)}$$

Due to the action of the gust the aircraft will tend to move upward; this vertical motion with speed w gives rise to a damping force

$$\Delta L_w = -\frac{1}{2}\rho V^2 S \frac{dC_L}{d\alpha}\frac{w}{V} \qquad \text{(A. 2)}$$

The equation of vertical motion of the aircraft reads

$$\frac{W}{g}\cdot\frac{dw}{dt} = \Delta L_g + \Delta L_w \qquad \text{(A. 3)}$$

With the dimensionless variable $s = \frac{V t}{\bar{c}}$ and the dimensionless "mass ratio parameter"

$$\mu_g = \frac{W/S}{\frac{1}{2}\rho g \bar{c}\frac{dC_L}{d\alpha}}$$

eq. (A. 3) can be written as

$$\mu_g \frac{dw}{ds} + w = U \qquad \text{(A. 4)}$$

With the initial condition: s = 0, w = 0, the solution is found:

$$\frac{dw}{ds} = \frac{U}{\mu_g} e^{-\frac{s}{\mu_g}} \qquad \text{(A. 5)}$$

The incremental load factor Δ n, which is equal to

$$\frac{1}{g}\frac{dw}{dt} = \frac{V}{g\bar{c}}\frac{dw}{ds},$$

s equal to Δ n =

$$= \frac{UV}{\mu_g g \bar{c}} e^{-\frac{s}{\mu_g}} \qquad \text{(A. 6)}$$

The maximum incremental load factor occurs at s = 0 and is equal to

$$\Delta n_{max} = \frac{U.V}{\mu_g . g . \bar{c}} \qquad \text{(A. 7)}$$

which may be rewritten in the form as expressed in the "sharp-edge gust formula" (2)

$$\Delta n_{max} = \frac{1}{2}\rho\frac{dC_L}{d\alpha}\cdot\frac{S}{W}\cdot U.V. \qquad \text{(A. 7a)}$$

A. 2 Ramp type gust (fig. 2a)

Aerodynamic inertia neglected.

The gust velocity is given by:

$$u = \frac{U.s}{H} \quad (0 < s < H)$$

$$u = -U \quad (s \geq H) \qquad \text{(A. 8)}$$

The lift force due to the gust is equal to:

$$\Delta L_g = \frac{1}{2} \rho V^2 S \frac{dC_L}{d\alpha} \cdot \frac{u}{V} . \qquad \text{(A. 9)}$$

The vertical motion gives rise to a lift force

$$\Delta L_w = -\frac{1}{2} \rho V^2 S \frac{dC_L}{d\alpha} \frac{w}{V} . \qquad \text{(A. 10)}$$

The equation of vertical motion, which reads

$$\frac{W}{g} \frac{dw}{dt} = \Delta L_g + \Delta L_w , \qquad \text{(A. 10a)}$$

can be rewritten

$$\mu_g \frac{dw}{ds} + w = \frac{U.s}{H} \quad (0 < s < H)$$

$$\mu_g \frac{dw}{ds} + w = U \quad (s \geq H) \qquad \text{(A. 11)}$$

With the initial condition s = 0, w = 0, the solution reads

$$\frac{dw}{ds} = \frac{U}{H} \left\{ 1 - e^{-\frac{s}{\mu_g}} \right\} \quad 0 < s < H$$

$$\frac{dw}{ds} = \frac{U}{H} \left\{ e^{\frac{H}{\mu_g}} - 1 \right\} e^{-\frac{s}{\mu_g}} \quad s \geq H \qquad \text{(A. 12)}$$

The maximum acceleration occurs at s = H, yielding a maximum load factor increment

$$\Delta n_{max} = \frac{U V}{\mu_g g \bar{c}} \left\{ \frac{\mu_g}{H} \left(1 - e^{-\frac{H}{\mu_g}}\right) \right\} . \qquad \text{(A. 13)}$$

By comparison of eqs (A. 7) and (A. 13) it turns out that the presence of a gradient distance H results in an alleviation of the maximum load-factor increment by

$$F = \frac{\Delta n_{max} \text{ (finite gradient)}}{\Delta n_{max} \text{ (sharp-edge)}} = \frac{\mu_g}{H} \left\{ 1 - e^{-\frac{H}{\mu_g}} \right\} . \qquad \text{(A. 14)}$$

It should be noted that the alleviation factor F is a function only of mass ratio parameter μ_g and gradient distance H.

It may easily be verified that the same expression for F is found for a triangular gust shape (fig. 2c).

A. 3 Arbitrary gust shape.

Including aerodynamic inertia.

A sharp-edge gust U occurring at s = 0 gives rise to an incremental lift force

$$(\Delta L g)_s = \frac{1}{2} \rho V^2 S \frac{dC_L}{d\alpha} \cdot \frac{U}{V} \psi_g (s) \qquad \text{(A. 15)}$$

φ_g (s) is a dynamic lift function, which is often approximated by

$$\psi_g(s) = 1 - c_1 e^{-\lambda_1 s} - c_2 e^{-\lambda_2 s} . \qquad \text{(A. 16)}$$

The coefficients c_1, c_2, λ_1 and λ_2 in this expression are only dependent on aspect ratio.

In the case of a gust with an arbitrary profile u = u(s) the lift force due to the gust at the instant s is equal to

$$\Delta L g = \frac{1}{2} \rho V^2 S \frac{dC_L}{d\alpha} \left[\psi_g(s) \cdot u_{s=0} + \int_0^s \frac{du}{d\sigma} \psi_g (s-\sigma) d\sigma \right] . \qquad \text{(A. 17)}$$

The upward movement w(s) of the aircraft induced by the gust gives rise to a downward lift force (w = 0 at s = 0):

$$\Delta L_w(s) = -\frac{1}{2} \rho V^2 S \frac{dC_L}{d\alpha} \int_0^s \frac{dw}{d\sigma} \psi_w (s-\sigma) d\sigma . \qquad \text{(A. 18)}$$

The term φ_w in this equation is a dynamic lift function which is often approximated by

$$\psi_w(s) = 1 - c_3 e^{-\lambda_3 s} . \qquad \text{(A. 19)}$$

The coefficients c_3 and h_3 are again only a function of aspect ratio. The equation of motion, which can be written again as

$$\mu_g \frac{dw}{ds} = \Delta L_g(s) + \Delta L_w(s)$$

can generally only be solved by numerical methods.

A large number of such calculations have been carried out, assuming different gust-shapes and different gradient distances. (See for example refs 1 to 4 inclusive.)

Regarding the calculated maximum accelerations, it was found from all calculations that the gust alleviation factor could be expressed as a function of mass ratio parameter μ_g and gradient distance H only.[3]

The values of the alleviation factors found are of course dependent on the shape of the assumed gust.

As a matter of fact the influence of gust shape is remarkably small, as can be seen by comparing the results of ref. 3, based on the gust profile of fig. 2a with those of ref. 4, based on the gust profile of fig. 2b.

[3] Note: Because the dynamic lift functions are a function of aspect ratio, the alleviation factor is also slightly dependent on aspect-ratio. As sailplanes generally have very slender wings, this influence will be neglected in the present discussion.