

Optimum Flying Speed for Lee Wave Soaring

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The theory giving optimum cruising speeds during thermal flights is well-known. Long-distance flights starting from the Wairarapa district of New Zealand and using lee waves have been described in the issues of "Sailplane & Gliding" of April 1958 and February 1964. These have suggested a variation of the problem, not to give maximum ground speed, but to give minimum loss of height in flying across a gap.

The situation visualised (and experienced in the Wairarapa) is that an extensive gap exists along the length of a mountain range, and has to be crossed in order to reach the next region of rising air. In this gap downdraughts must be expected, although under the most favourable conditions air that is flowing horizontally might be found.

It is assumed that the glider pilot has climbed as high as possible in a standing wave at one end of the gap and is setting out to cross the gap. His problem is to select the best speed and it is supposed that the strengths of the downdraughts are unpredictable, but that the mean horizontal speed of the wind is known reasonably accurately from a meteorological forecast.

It is suggested that the speed selected should be that which will give minimum loss of height per distance covered across the gap. The pilot does not know for certain whether he will be able to cross the gap successfully—if the downdraughts prove to be unusually severe this may not be possible. But on the other hand if a successful crossing proves possible, arrival on the far side with minimum loss of height will place the sailplane in the strongest possible region of lift, since near the ground the strength of the lift due to a wave normally increases with height.

Theoretical Considerations

Let v = true airspeed of the sailplane
 s = $s(v)$ = true rate of sink of the sailplane at speed v
 k = horizontal component of the wind speed
 g = ground speed achieved across the gap
 u = strength of the downdraught in the gap
 H = height lost during the traverse of a length L of the gap.

Assume that the speeds and distances involved are measured in consistent units.

The track to be followed by the sailplane is assumed to be the line joining the waves at each end of the gap, and obviously an into-wind component of speed equal to the wind-strength must be maintained to prevent drifting off-track.

By the theorem of Pythagoras

$$g^2 = v^2 - k^2$$

Time to cross a length L of the gap = L/g so that height lost,

$$H = \frac{(u+s)L}{(v^2 - k^2)^{1/2}}$$

$$\frac{dH}{dv} = \frac{(v^2 - k^2)s' - v(u+s)}{(v^2 - k^2)^{3/2}} L$$

Thus for minimum loss of height, v must be chosen so that

$$s' = \frac{v(u+s)}{(v^2 - k^2)} \quad (1)$$

This condition does not lend itself to a graphical interpretation (as in the thermal soaring case), but can be applied numerically when the performance curve of the sailplane is known.

According to Welch & Irving (Ref. 1), the polar of a sailplane may be taken to be

$$s = k_1 v^3 + k_2/v$$

in which case condition (1) leads to an equation of the sixth degree. However the Welch- Irving formula does not give a perfect fit to a measured polar, so that numerical solution is to be preferred to an algebraic one.

Once the optimum speed, v_0 , for minimum height loss according to (1) has been determined, the expression $(u + s)/g$ gives the ratio of the height lost to the distance travelled across the gap, so that estimates can be made of the height required to cross any particular size of gap successfully.

If the wind varies considerably with altitude, the procedure would be to adjust the speed to correspond to the wind forecast for the appropriate altitude. Convenient methods of presentation of the results readily suggest themselves for the various types of variometer.

For use at great altitudes correction for density effects should be made along the lines suggested by MacCready in "Soaring", March-April, 1954.

Computations for Slingsby Skylark 3F

As an example to illustrate the values involved, calculations have been made for the Skylark 3F, number RAF 146, using the polar of Reference (2):

Wind-speed k in knots	Optimum speed v_0 in knots	Height lost in feet per N. M. of gap		
		at v_0	at $v_0 + 5$	at $v_0 - 5$
1. Zero downdraught ($u=0$)				
60	75	630	645	660
45	58	398	408	425
30	47	260	270	270
15	43	200	214	226
2. Downdraught of 3 knots				
60	87.5	965	975	978
45	73.5	779	783	790
30	63	655	660	662
15	57	587	590	597
3. Downdraught of 6 knots				
60	97	1230	1236	1238
45	84	1063	1067	1071
30	75	943	951	954
15	70	880	884	885

The effects of a 5 knot divergence from the optimum speed are shown. It will be seen that in general the penalty for flying a little faster than the optimum speed is slightly smaller than the penalty for flying a little more slowly than the optimum speed.

For practical application these results have been put into graphical form, from which the following table has been prepared for use in conjunction with a variometer reading in knots:

Variometer reading in knots	Wind 15 knots	Range	Wind 30 knots	Range	Wind 45 knots	Range	Wind 60 knots	Range
12	75	1.00	78	0.98	83	0.95	91	0.93
9	67	1.19	71	1.17	77	1.13	85	1.10
6	59	1.56	63	1.51	70	1.46	79	1.40
3	50	2.60	53	2.40	60	2.17		

The variometer reading is in knots (down-wards), the wind-speed and v_0 in knots and the range is in nautical miles per 1000' of altitude.

Optimum Cruising Speeds

In calculating optimum cruising speeds in the case of thermal soaring it is assumed that if a certain rate of climb has been achieved in one thermal, a similar rate of climb might be expected in the next. The corresponding assumption in the case of flights along a series of lee waves seems more difficult to justify, as the lift in the wave on one side of a gap need not be closely related to that on the other side of the gap. However it is instructive to investigate the case when similar rates of climb can be expected on the two sides of the gap.

Let c = rate of climb achieved in the lee wave. The time taken to regain the height lost in crossing the length L of the gap = $L(u + s)/gc$.

Total time to cover distance $L = L(c + u + s)/gc$.

Average speed =

$$\frac{c(v^2 - k^2)^{1/2}}{c + u + s}$$

This is a maximum when $s' =$

$$\frac{(c + u + s)v}{v^2 - k^2} \quad (4)$$

Comparison with result (1) shows that the calculations already made can be adapted to cover this case, simply by replacing the u in (1) by $c + u$.

The above analysis suffers from the disadvantage that it assumes that all the height is regained while the sailplane remains over one spot, whereas in practice it is often possible to regain height in a wave while continuing to cover ground parallel to the mountain range.

Nevertheless, disregarding this possibility, it is interesting to note that the optimum speed for use across a gap in which there is a downdraught u when it is desired to minimize the loss of height, is the same as the best cruising speed across a gap when the sum of the downdraught speed and the achieved rate of climb is equal to u .

References:

1. Welch and Irving. "The Soaring Pilot". John Murray. 1955, Chapter 3.
2. Doetsch and Lampard. "Sailplane & Gliding". Feb.-Mar. 1965, p. 44.