

# Spanwise Distribution of Aerodynamic Shear and Bending-Moment on Cantilever Tapered Wings

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## Summary

The spanwise air-load distribution for wings with twist is obtained by Schrenk's approximation method. Then the distributions of shear and bending-moment due to the air load are calculated, for the case of a wing having a parallel centre section and trapezoidal outer parts with twist on the latter only and being of the usual form involving straight leading and trailing edges. The shear and bending-moment diagrams are prepared for use in the structural design of sailplane wings.

## Approximate Lift Distribution for Tapered and Twisted Wings

The lift distribution for wings with aerodynamic twist may be considered to comprise two parts, the basic lift distribution due to twist, and the additional lift distribution for the wing with no twist. In his approximate method for obtaining the spanwise lift distribution, Schrenk assumed that for wings with no twist and constant wing section, the additional lift distribution is the average between an elliptical distribution and the planform distribution (see Fig. 1). This approximation has been accepted generally as satisfactory for the airplane structural design work.

Assuming a constant slope of the section lift coefficient curve along the span, the additional lift distribution corresponding to  $C_L = 1.0$  is therefore obtained from the equation:

$$c c_{la1} = \frac{1}{2} \left\{ c + \frac{4S}{\pi b} \sqrt{1 - \eta^2} \right\} \quad (1)$$

where  $c$  is the wing chord (m) at a point on the span,  $c_{la1}$  the additional section lift coefficient corresponding to  $C_L = 1.0$  for the entire wing,  $S$  the wing area ( $m^2$ ),  $b$  the span (m), and  $\eta$  the spanwise station or section expressed as a fraction of the semi-span.

When the slope  $a_0$  of the section lift coefficient curve is constant along the span, the angle  $a_0$  (deg) from the zero-lift chord of the root section to the plane of zero-lift for the entire wing is found from the equation

$$\alpha_0 = \frac{b}{S} \int_0^1 e c d\eta \quad (2)$$

where  $e$  is the angle (deg) between the zero-lift chord of the root section and that of  $\eta$  section, i. e. the aerodynamic twist angle at  $\eta$  section.

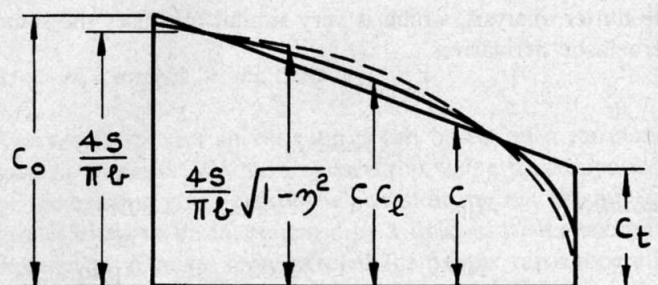


FIG. 1. SCHRENK CONSTRUCTION

$a_a$ , the angle of attack (deg) measured from the zero-lift plane of the entire wing to the zero-lift chord line of the section is

$$\alpha_a = e - \alpha_o \quad (3)$$

$a_e$ , the effective angle of attack (deg) of the section, for the zero-lift condition of the entire wing is

$$a_e = a_a - a_i$$

where  $a_i$  is the induced angle of attack. Schrenk assumed that  $a_i = 0.5 a_a$  and hence that  $a_e = 0.5 a_a$  also. Here using the relations  $dC_L/da = 2\pi A/(A+2)$ , and  $a_i = 2C_L/\pi A = 4a_a/(A+2)$  where  $A$  is the aspect ratio,

$$\begin{aligned} \text{we find that } a_e &= \left(1 - \frac{a_i}{a_a}\right) a_a \\ &= \left(\frac{A-2}{A+2}\right) a_a \\ &= K a_a \end{aligned}$$

where  $K$  is a function only of aspect ratio.

The basic lift distribution is obtained from the equation

$$cc_{1b} = ca_o \alpha_e = ca_o k \alpha_a \quad (5)$$

where  $a_o$  (1/deg) is the slope of the section lift curve.

Using Eqs. (1) and (5), the total lift distribution corresponding to any value of  $C_L$  may be found as

$$cc_1 = C_L cc_{1a1} + cc_{1b} \quad (6)$$

It is convenient to use the term  $cc_1$  which multiplied by the dynamic pressure  $q$ , will give the air load per unit span.

For the wing shown in Fig. 2, having the central fraction  $\eta$  of the span rectangular and the remainder trapezoidal, the chord lengths  $c_o$ ,  $c$ , and  $c_t$  (m) can be expressed as

$$\begin{aligned} c_o &= \frac{2S}{b} \left\{ \frac{1}{(1+a)+(1-a)\lambda} \right\} \\ c &= \frac{2S}{b} \left\{ \frac{(1-\eta)+(2-a)\lambda}{(1-a^2)+(1-a)^2\lambda} \right\} \\ c_t &= \frac{2S}{b} \left\{ \frac{\lambda}{(1+a)+(1-a)\lambda} \right\} \end{aligned} \quad (7)$$

where  $\lambda = c_t/c_o =$  wing taper ratio.

The additional lift distribution corresponding to  $C_L = 1.0$  may be found from Eqs. (1) and (7) as follows: for rectangular part

$$cc_{1a1} = \frac{S}{b} \left\{ \frac{1}{(1+a)+(1-a)\lambda} + \frac{2}{\pi} \sqrt{1-\lambda^2} \right\}$$

for trapezoidal part

$$cc_{1a1} = \frac{S}{b} \left\{ \frac{(1-\eta)+(2-a)\lambda}{(1-a^2)+(1-a)^2\lambda} + \frac{2}{\pi} \sqrt{1-\lambda^2} \right\} \quad (8)$$

It is assumed that only the outer trapezoidal parts of the wing are twisted in such a way that both leading and trailing edges of the trapezoidal parts make straight lines. The aerodynamic twist  $e$  (deg) at  $\eta$  section of the tapered part against the inner rectangular part will be

$$e = \frac{(\eta-a)\lambda}{(1-\eta)+(2-a)\lambda} (e_t) \quad (9)$$

where  $e_t$  is the aerodynamic twist angle (deg) at the wing tip relative to the rectangular part.

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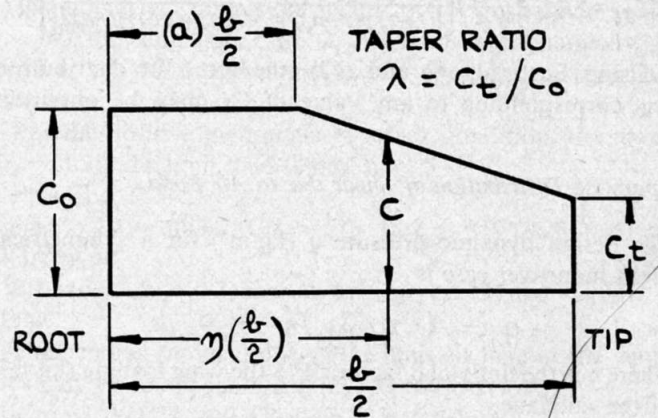


FIG. 2 WING NOMENCLATURE

The angle  $\alpha_o$  (deg) measured from the zero-lift chord of the midspan section to the plane of zero-lift for the entire wing will be found from Eqs. (2) and (9) as follows:

$$\alpha_o = \frac{(1-a)\lambda}{(1+a)+(1-a)\lambda} (e_t) \quad (10)$$

From Eqs. (3), (9) and (10), the angle  $a_a$  (deg) measured from the zero-lift plane of the entire wing to the zero-lift chord line of the section will be: for rectangular part

$$\alpha_a = \frac{(1-a)\lambda}{(1+a)+(1-a)\lambda} (e_t)$$

for trapezoidal part

$$\alpha_a = \left\{ \frac{(1-a)\lambda}{(1+a)+(1-a)\lambda} - \frac{(\eta-a)\lambda}{(1-\eta)+(2-a)\lambda} \right\} (e_t) \quad (11)$$

The basic lift distribution will be obtained from Eqs. (5) and (11).

for rectangular part

$$cc_{1b} = 2ka_o e_t \left(\frac{S}{b}\right) \left\{ \frac{(1-a)\lambda}{(1+a)+(1-a)\lambda} \right\}^2$$

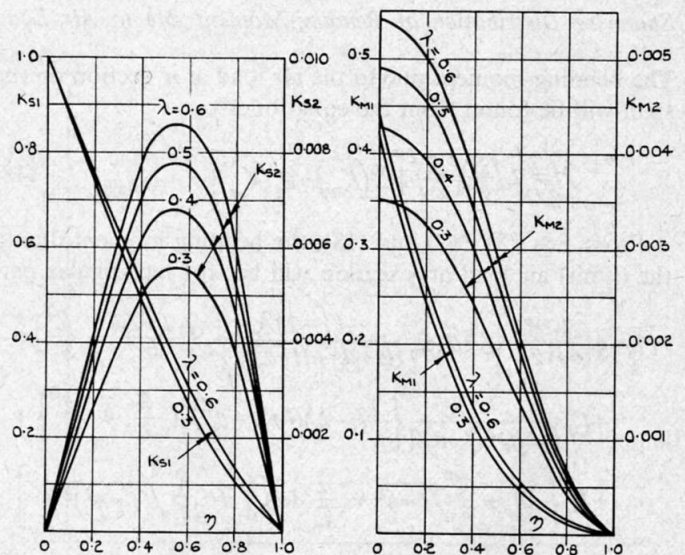


FIG. 3 SHEAR FORCE & BENDING MOMENT FUNCTIONS  $a = 0.3$

for trapezoidal part (12)

$$C_{L_{1/8}} = 2ka_0 e_t \left( \frac{S}{b} \right) \left[ \frac{(1-\eta) + (2-a)\lambda}{\{(1+a) + (1-a)\lambda\}^2} - \frac{(2-a)}{(1-a)\{(1+a) + (1-a)\lambda\}} \right] \lambda$$

Using Eqs. (6), (8) and (12), the total lift distribution  $C_{L_{1/8}}$  corresponding to any value of  $C_L$  may be obtained.

#### Spanwise Distribution of Shear due to Air Load

The design dynamic pressure  $q$  (kg/m<sup>2</sup>) for a symmetrical flight maneuver case is

$$q = (n/C_L)(W/S) \quad (13)$$

where  $n$  is the limit load factor,  $W/S$  the wing loading (kg/m<sup>2</sup>) of the sailplane.

The shear due to the air load at  $\eta'$  section on the span is

$$S_{\eta'} = q \left( \frac{S}{2} \right) \int_{\eta}^1 C_{L_{1/8}} d\eta \quad (14)$$

From Eqs. (8), (12) and (14), the shear due to the air load at  $\eta$  section on the span will be: for rectangular part

$$S_{\eta} = q \left( \frac{S}{2} \right) \left[ -2ka_0 e_t \left\{ \frac{(1-a)\lambda\eta}{\{(1+a) + (1-a)\lambda\}^2} + C_L \left\{ \frac{-\eta}{(1+a) + (1-a)\lambda} + 1 - \frac{\eta}{\pi\sqrt{1-\eta^2}} - \frac{1}{\pi} \sin^{-1} \eta \right\} \right] \right] \quad (15)$$

for trapezoidal part

$$S_{\eta} = q \left( \frac{S}{2} \right) \left[ 2ka_0 e_t \frac{(a^2 - \eta)(1-\eta)\lambda}{(1-a)\{(1+a) + (1-a)\lambda\}^2} + C_L \left\{ \frac{(1-\eta)}{(1-a) + (1-a)\lambda} \left\{ (1-a)\lambda - \frac{1}{2}(1-\eta)(1+\eta) \right\} + \frac{1}{2} - \frac{\eta}{\pi\sqrt{1-\eta^2}} - \frac{1}{\pi} \sin^{-1} \eta \right\} \right] \quad (16)$$

Putting  $a = 0$  in Eq. (16), the shear  $S_{\eta}$  for a twisted trapezoidal wing will be:

$$S_{\eta} = q \left( \frac{S}{2} \right) \left[ -2ka_0 e_t \frac{\eta(1-\eta)\lambda}{(1+\lambda)^2} + C_L \left\{ \frac{-\eta}{(1+\lambda)} + \frac{(1-\eta)\eta^2}{2(1+\lambda)} + 1 - \frac{\eta}{\pi\sqrt{1-\eta^2}} - \frac{1}{\pi} \sin^{-1} \eta \right\} \right] \quad (17)$$

#### Spanwise Distribution of Bending-Moment due to Air Load

The bending-moment due to the air load at  $\eta'$  section on the span will be found from the equation:

$$M_{\eta'} = q \left( \frac{S}{2} \right) \left( \frac{S}{2} \right) \left[ C_L (K_{m1}) - e_t (K_{m2}) \right] \quad (\text{kg.m}) \quad (18)$$

From Eqs. (8), (12) and (18), the bending moment due to the (limit) air load at  $\eta$  section will be: for rectangular part

$$M_{\eta} = q \left( \frac{S}{2} \right) \left( \frac{S}{2} \right) \left[ 2ka_0 e_t \frac{-(1-a)\lambda}{\{(1+a) + (1-a)\lambda\}^2} \left( \frac{1+2a}{6} - \frac{\eta^2}{2} \right) + C_L \frac{1}{(1+a) + (1-a)\lambda} \left\{ \frac{1}{6}(1-\lambda)(1+a+a^2) + \frac{\lambda}{2} + \frac{\eta^2}{2} \right\} + C_L \left\{ -\eta + \frac{\eta^2}{\pi\sqrt{1-\eta^2}} + \frac{\eta}{\pi} \sin^{-1} \eta + \frac{2}{3\pi} \sqrt{(1-\eta^2)^3} \right\} \right] \quad (19)$$

for trapezoidal part

$$M_{\eta} = q \left( \frac{S}{2} \right) \left( \frac{S}{2} \right) \left[ 2ka_0 e_t \frac{-\lambda}{(1-a)\{(1+a) + (1-a)\lambda\}^2} \left\{ \frac{1}{6}(1-3a^2+2\eta)(1-\eta)^2 \right\} + C_L \frac{1}{(1-a^2) + (1-a)\lambda} (1-\eta)^2 \frac{1}{6} \left\{ 1 + (2-3a)\lambda - (1-\lambda)\eta \right\} + C_L \left\{ -\frac{\eta}{2} + \frac{\eta^2}{\pi\sqrt{1-\eta^2}} + \frac{\eta}{\pi} \sin^{-1} \eta + \frac{2}{3\pi} \sqrt{(1-\eta^2)^3} \right\} \right] \quad (20)$$

Putting  $\eta = 0$  in Eq. (19), the bending-moment  $M_0$  at the center of the span, or the wing root will be:

$$M_0 = q \left( \frac{S}{2} \right) \left( \frac{S}{2} \right) \left[ 2ka_0 e_t \frac{-\lambda}{\{(1+a) + (1-a)\lambda\}^2} \left\{ \frac{1}{6}(1-a)(1+2a) \right\} + C_L \frac{1}{(1+a) + (1-a)\lambda} \left\{ \frac{1}{6}(1-\lambda)(1+a+a^2) + \frac{\lambda}{2} \right\} + C_L \left( \frac{2}{3\pi} \right) \right] \quad (21)$$

Putting  $a = 0$  in Eq. (20), the bending-moment  $M_{\eta}$  due to the (limit) air load for a twisted trapezoidal wing will be: (22)

$$M_{\eta} = q \left( \frac{S}{2} \right) \left( \frac{S}{2} \right) \left[ ka_0 e_t \frac{-\lambda}{3(1+\lambda)^2} (1+2\eta)(1-\eta)^2 + C_L \frac{(1-\eta)^2}{6(1+\lambda)} \left\{ 1 + 2\lambda - (1-\lambda)\eta \right\} + C_L \left\{ \frac{-\eta}{2} + \frac{\eta^2}{\pi\sqrt{1-\eta^2}} + \frac{\eta}{\pi} \sin^{-1} \eta + \frac{2}{3\pi} \sqrt{(1-\eta^2)^3} \right\} \right]$$

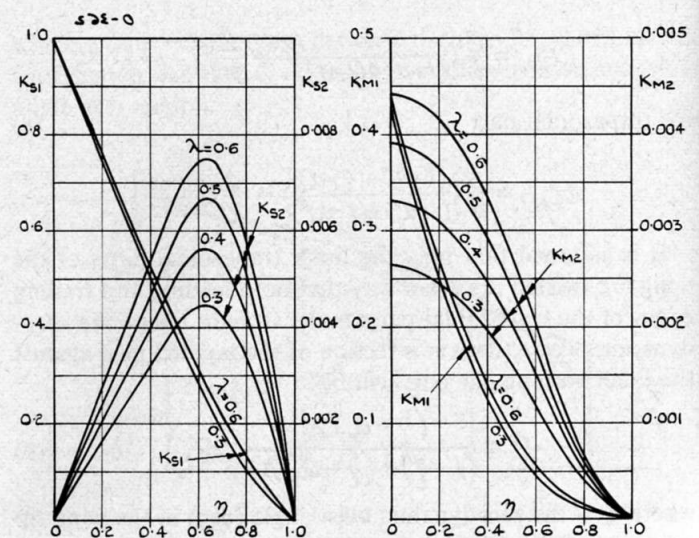
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#### Shear and Bending-Moment Diagrams for Tapered and Twisted Wings

The coefficient ( $k$ ) in the shear equations (15) and (16) is given by Eq. (4) as  $k = (A-2)/(A+2)$ , where  $A$  is the wing aspect ratio. For  $A = 14, 18$  and  $25$ ,  $k = 0.75, 0.80$  and  $0.85$  respectively, i.e. never departs very much from 0.8. Also the slope  $a_0$  of the section lift coefficient curve may be considered in many cases as 0.11 (1/deg).

Now the shear equations (15) and (16) may be written in the following form:

$$S_{\eta} = q \frac{S}{2} \left[ C_L (K_{S1}) - e_t (K_{S2}) \right] \quad (\text{kg}) \quad (23)$$



where  $K_s$  and  $K_{s2}$  are functions of  $a$ ,  $\lambda$  and  $\eta$ . They are plotted in Figs. 3 and 4 for values of 0,3 and 0,5 and of from 0,3 to 0,6.

Also the bending-moment equations (19) and (20) may be written in the following form:

$$M_\eta = q \left( \frac{b}{2} \right)^2 \int_{\eta_1}^{\eta_2} C C_x (\eta - \eta_1) d\eta \quad (\text{kg}\cdot\text{m}) \quad (24)$$

where  $K_{m1}$  and  $K_{m2}$  are also functions of  $a$ ,  $\lambda$  and  $\eta$ , and again are plotted in Figs. 3 and 4 for the same ranges of values.

*Example.* Consider the bending-moment due to the limit air load on the wing of Skylark-3F sailplane, for the positive high angle of attack condition.

The wing area,  $S = 16.1 \text{ (m}^2\text{)}$ , the wing span,  $b = 18.2 \text{ (m)}$ , the gross weight,  $W = 359 \text{ (kg)}$

The coefficient giving the length of the rectangular part  $a = 0.3$ , the wing taper ratio,  $\lambda = 0.5$ , the aerodynamic twist angle at the wing tip relative to the untwisted rectangular part,  $e_t = 3.0 \text{ (deg)}$ .

For the positive high angle of attack condition, the max.  $C_L = 1.28$ , the limit load factor,  $n = 5$

From Eq. (13)  
 $q(S/b) (b/2)^2 = nbW/4C_L = 5 \times 18.2 \times 359 / (1.28 \times 4) = 6380 \text{ (kg}\cdot\text{m)}$ .

From fig. 3 ( $a = 0.3$ ) for  $\eta = 0$ ,  $K_{m1} = 0.434$  and  $K_{m2} = 0.0049$ .

The bending-moment due to the limit air load at the wing root may be found from Eq. (24) as follows:

$$M_0 = 6380 (1.28 \times 0.434 - 3 \times 0.0049) = 6380 (0.555 - 0.015) = 3540 - 95 = 3445 \text{ (kg}\cdot\text{m)}$$

## Loads on Gliders in Turbulent Atmosphere

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### Introduction

This paper presents some statistical data collected since 1956 at the Chair of Mechanics, Department of Power and Aircraft Engineering, Warsaw Technical University. The first results, concerning some 850 test hours have already been published in the papers of the IX OSTIV Congress (3). The present data concern a period of about 1600 hours.

In addition the paper contains a brief discussion of a certain mathematical scheme describing in an approximate manner a load spectrum.

### The Scheme of a Load Spectrum

This scheme has been devised following the idea of H. Press (2), who makes use of the results of S. O. Rice (1).

To explain the essentials of this scheme we shall consider a simpler problem and show how to pass from it to the spectrum proper.

The scheme under consideration is composed of "elements" which are distinguished from one another by the symbol  $\sigma_i$ . The occurrence of an overload  $a_n$  in the element  $\sigma_i$  is a random event per unit time with a Gaussian type distribution of probability. It can, therefore, be said that the probability of overload  $a_n$  in the "element  $\sigma_i$ " per unit time is  $P^{\sigma_i}(a_n)$  from the Gaussian probability function. The probability of meeting the "element  $\sigma_i$ " in a given period of time of unit length is  $P(\sigma_i)$  (shown in Fig. 1) therefore the

probability of meeting the overload  $a_n$  in that period of time as a result of meeting the "element  $\sigma_i$ " is expressed by the product of probabilities:

$$P^{\sigma_i}(a_n) \cdot P(\sigma_i)$$

and may be called the conditional probability of meeting the overload  $a_n$  on condition of meeting the "element  $\sigma_i$ ".

It is intuitively obvious that the cumulative probability of meeting a given  $a_n$  in flight during a unit period of time will be composed of all the possible conditional probabilities, it is therefore expressed by the sum:

$$P^{\sigma_1}(a_n) \cdot P(\sigma_1) + P^{\sigma_2}(a_n) \cdot P(\sigma_2) + \dots + \dots = \sum_i P^{\sigma_i}(a_n) \cdot P(\sigma_i) = M(a_n)$$

This is the result for the "simpler variant". In the load spectrum proper the random event of "occurrence of  $a_n$ " is represented by an interval of the random function, of unit length, constituting an expression of the load process during

Fig. 1.

