where K_s and K_{s2} are functions of a, λ and η. They are plotted in Figs. 3 and 4 for values of 0,3 and 0,5 and of from 0,3 to 0,6.

Also the bending-moment equations (19) and (20) may be written in the following form:

$$M_\eta = q\left(\frac{b}{2}\right)^2 \int_{\eta'}^{1} c\, c_l (\eta - \eta')\, d\eta \qquad \text{(kg.m)} \qquad (24)$$

where K_{m1} and K_{m2} are also functions of a, λ and η, and again are plotted in Figs. 3 and 4 for the same ranges of values.

Example. Consider the bending-moment due to the limit air load on the wing of Skylark-3F sailplane, for the positive high angle of attack condition.

The wing area, S = 16.1 (m^2), the wing span ,b= 18.2 (m), the gross weight, W = 359 (kg)

The coefficient giving the length of the rectangular part a = 0.3, the wing taper ratio, $\lambda = 0.5$, the aerodynamic twist angle at the wing tip relative to the untwisted rectangular part, $e_t = 3.0$ (deg).

For the positive high angle of attack condition, the max. $C_L = 1.28$, the limit load factor, n = 5

From Eq. (13)

$q(S/b)(b/2)^2 = nbW/4C_L = 5 \times 18.2 \times 359/(1.28 \times 4) = 6380$ (kg.m).

From fig. 3 (a = 0.3) for η = o, $K_{m1} = 0.434$ and $K_{m2} = 0.0049$.

The bending-moment due to the limit air load at the wing root may be found from Eq. (24) as follows:

$M_o = 6380\,(1.28 \times 0.434 - 3 \times 0.0049)$

$= 6380\,(0.555 - 0.015) = 3540 - 95 = 3445$ (kg.m)

Loads on Gliders in Turbulent Atmosphere

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Introduction

This paper presents some statistical data collected since 1956 at the Chair of Mechanics, Department of Power and Aircraft Engineering, Warsaw Technical University. The first results, concerning some 850 test hours have already been published in the papers of the IX OSTIV Congress (3). The present data concern a period of about 1600 hours.

In addition the paper contains a brief discussion of a certain mathematical scheme describing in an approximate manner a load spectrum.

The Scheme of a Load Spectrum

This scheme has been devised following the idea of H. Press (2), who makes use of the results of S. O. Rice (1).

To explain the essentials of this scheme we shall consider a simpler problem and show how to pass from it to the spectrum proper.

The scheme under consideration is composed of "elements" which are distinguished from one another by the symbol σ_i. The occurence of an overload a_n in the element σ_i is a random event per unit time with a Gaussian type distribution of probability. It can, therefore, be said that the probability of overload a_n in the "element σ_i" per unit time is $P^{\sigma_i}(a_n)$ from the Gaussian probability function. The probability of meeting the "element σ_i" in a given period of time of unit length is $P(\sigma_i)$ (shown in Fig. 1) therefore the probability of meeting the overload a_n in that period of time as a result of meeting the "element σ_i" is expressed by the product of probabilities:

$$P^{\sigma_i}(a_n) \cdot P(\sigma_i)$$

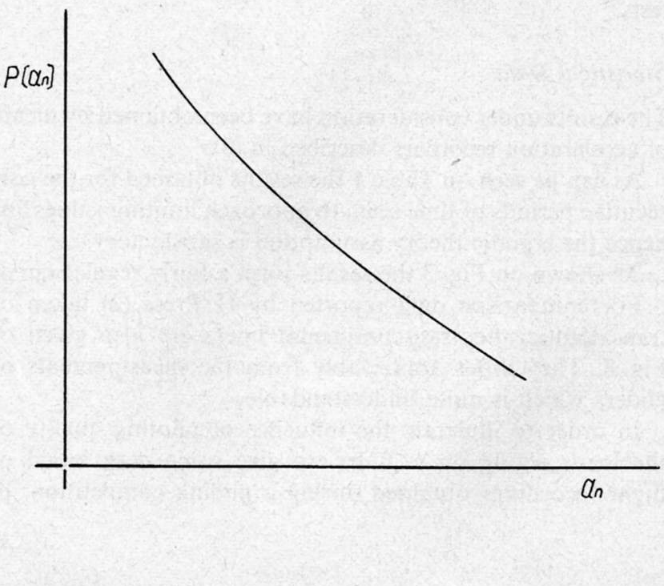
and may be called the conditional probability of meeting the overload a_n on condition of meeting the "element σ_i".

It is intuitively obvious that the cumulative probability of meeting a given a_n in flight during a unit period of time will be composed of all the possible conditional probabilities, it is therefore expressed by the sum:

$$P^{\sigma_1}(a_n) \cdot P(\sigma_1) + P^{\sigma_2}(a_n) \cdot P(\sigma_2) + \ldots + \ldots = \sum_i P^{\sigma_i}(a_n) \cdot P(\sigma_i) = M(a_n)$$

This is the result for the "simpler variant". In the load spectrum proper the random event of "occurence of a_n" is represented by an interval of the random function, of unit length, constituting an expression of the load process during

Fig. 1.



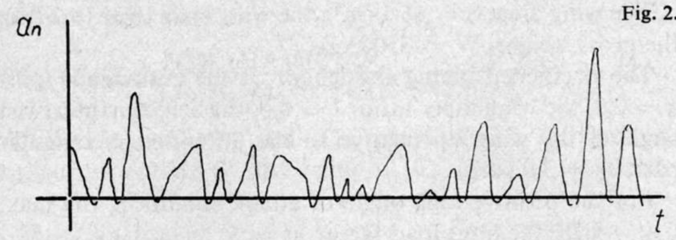


Fig. 2.

a hypothetical flight within the fundamental "element σ_i". An exemple of such an expression is given in Fig. 2. The random function considered has the following properties:

a) there exists a mean value (the process is stationary)
b) the mean square overload σ^2 is constant
c) the probability distribution of overloads is of the Gaussian type

To each "element σ_i" is attached a definite probability in the form of the function $P(^{\sigma}i)$. The term $P^{\sigma i}(a_n)$ expressing the probability of occurence of the random event under consideration is replaced by the expected number of maxima above the line a_n, attained by the random function in the interval under consideration. In other words, it is the most expected number of times with which a given overload level will be exceeded per unit time. This quantity is expressed by the equation:

$$N^{\sigma i}(a_n) = N_o \exp\left[-\frac{a_n^2}{2\sigma_i^2}\right]$$

The coefficient N_o is constant and expresses the expected number of zeros of the random function under consideration in an interval of unit length.

Let us observe next, that the probability $P(^{\sigma}i)$ can be interpreted as the time (fraction of unit time) during which the glider remains within the "element σ_i". Thus, the product:

$$N^{\sigma i}(a_n) \cdot P(\sigma i)$$

can be interpreted as the number of times with which the level has been exceeded as a result of meeting the "element σ_i". In order to find the representation of the cumulative probability of meeting the given overload a_n we must perform summation over all "elements σ_i". We obtain:

$$M(a_n) = \sum_i N^{\sigma}(a_n) \cdot P(\sigma i) = N_o \sum_i P(\sigma i)\, e^{-\frac{a_n^2}{2\sigma i^2}}$$

This is the result. The left-hand member of this equation can be identified, in agreement with the ergodic theory with the observed frequencies obtained as a result of long duration tests.

Statistical Data

The results under consideration have been obtained by means of acceleration recorders described in (3).

As can be seen on Table 1 the results obtained for the consecutive periods of time seem to approach limiting values and hence the ergodic theory assumption is satisfactory.

As shown on Fig. 3 the results form a fairly regular curve.

For comparison data reported by H. Press (2) taken on transatlantic and transcontinental liners are also given on Fig. 3. They differ appreciably from the measurements on gliders which is quite understandable.

In order to illustrate the influence of piloting quality on the loads acting on a glider we give some data based on flight recordings obtained during a gliding competition, by

($\overline{M(a_n)}$)

		1.7g	2.0g	2.5g	3.0g	3.5g	4.0g	5.0g
284 h	1956 - 1.03.1962	1.39×10^{-3}	4.0×10^{-4}	1.28×10^{-4}	4.70×10^{-6}	1.76×10^{-5}	5.82×10^{-6}	–
446 h	1.03.1962 - 30.10.1962	1.48×10^{-3}	3.93×10^{-4}	0.87×10^{-4}	2.25×10^{-5}	0.87×10^{-5}	3.72×10^{-6}	6.22×10^{-7}
856 h	30.10.1962 - 24.07.1964	1.90×10^{-3}	6.71×10^{-4}	1.76×10^{-4}	4.70×10^{-5}	1.72×10^{-5}	8.21×10^{-6}	11.4×10^{-7}
1586 h	1956 - 24.07.1964	1.69×10^{-3}	5.41×10^{-4}	1.46×10^{-4}	4.00×10^{-5}	1.49×10^{-5}	6.48×10^{-6}	7.85×10^{-7}

Table 1. Warsaw Technical University Statistics

a pilot known for his particularly rough piloting technique. Although the number of recording hours was very small these data are consistent over the entire range of recorded loads.

For comparison we show the results concerning an average flight performance obtained also during a gliding competition. They are grouped in the neighbourhood of the average data for entire period of 1600 hours.

We give also some approximate data for average flights. Despite the small number of recording hours the specific character of the loads can be observed, different from that of an average thermal flight under the clouds. The data are close to the rough piloting results.

We have not succeded in showing the differences connected with the type of glider. The above results have been obtained with gliders of similar dynamic characteristics. The differences, which are small, are presumably affected to a large extent by the piloting technique, as illustrated by the above quoted examples.

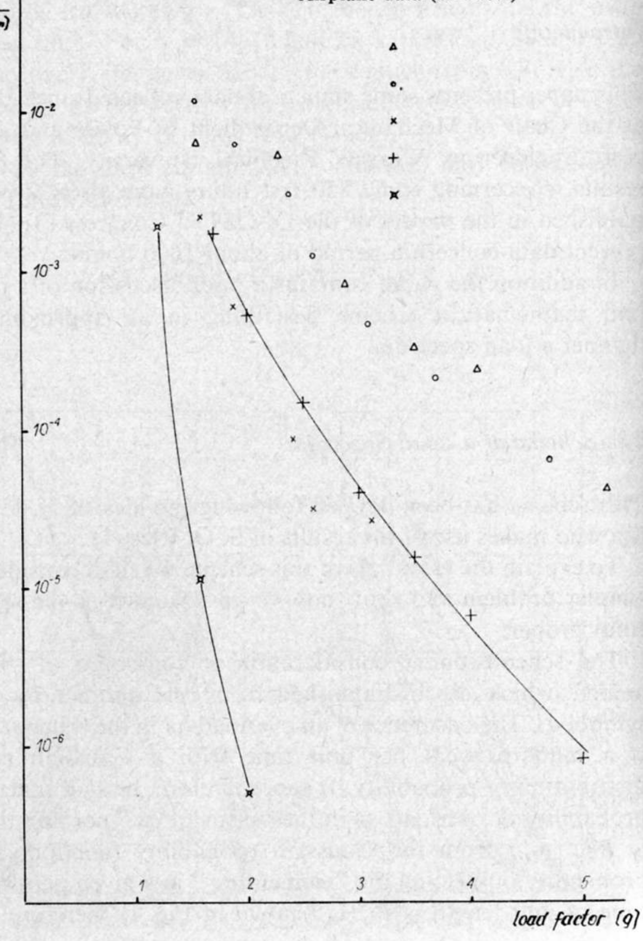


Fig. 3. △ Terminal flight with rough piloting
○ Mountain wave flight
× Flight during a soaring competition
+ Summary of all records (1586 hours)
* Airplane data (H. Press)

		$\overline{M(a_n)}$							
		1.3	1.6	2.0	2.5	3.0	3.5	4.0	4.5
12h	thermal flight with rough piloting	–	6.0×10^{-3}	6.0×10^{-3}	2.4×10^{-3}	6.5×10^{-4}	3.11×10^{-4}	2.3×10^{-4}	7.6×10^{-5}
30h	flight during a soaring competition	–	2.26×10^{-3}	1.3×10^{-4}	5.7×10^{-5}	2.5×10^{-5}	–	–	–
12h	„wave mountain" flight	–	11.2×10^{-3}	5.4×10^{-3}	2.2×10^{-3}	5.9×10^{-4}	2.7×10^{-4}	1.13×10^{-4}	7.5×10^{-5}
7224h	H. Press data for airplanes	1.9×10^{-3}	2.0×10^{-5}	5.0×10^{-7}	–	–	–	–	–

Table 2. Comparison of Soaring and Airplane Statistics

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(1) *S. O. Rice:* "Mathematical Analysis of Random Noise", Bell Syst. Tech. Jour. vol. 23 No. 3, vol. 24 No. 1 (1944–45).

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