On the Dynamic Response of Sailplanes to Longitudinal Manoeuvres

By Piero Morelli

1 Introduction

The aim of the present study is to show how the dynamic response of a sailplane to elevator control in a longitudinal manoeuvre differs from the typical one of other types of aircraft.

The response of sailplanes in most cases, is virtually not oscillatory but aperiodic. If the case of an unchecked manoeuvre is considered, the maximum value of the instantaneous normal acceleration resulting from the manoeuvre practically coincides with the asymptotic value, relating to the steady state of accelerated flight corresponding to the final new static equilibrium condition.

The expressions which define the dependence of load factors and tail loads from the elevator deflection are developed, as a function of the relevant aerodynamic and inertial parameters of the aircraft.

An analysis of this expression suggests a revision, and possible modification, of present airworthiness requirements concerning tail manoeuvring loads.

2 *Theoretical Analysis*

The equations of motion of an aircraft in symmetric manoeuvres, referred to Eulerian axes fixed with the moving aircraft, can be written as follows:

$$\frac{W}{g}\left(\frac{dw}{dt} - qV\right) + \frac{\partial C_L}{\partial \alpha}\alpha S \frac{1}{2}\rho V^2 = 0 \qquad (1)$$

$$J_y \frac{dq}{dt} - \frac{\partial C_m}{\partial \alpha}\alpha S c \frac{1}{2}\rho V^2 - \frac{\partial C_m}{\partial q} q S c \frac{1}{2}\rho V^2 + \frac{\partial C_{Lt}}{\partial \alpha_t}\frac{d\alpha}{dt}\frac{l_t}{V}\frac{d\varepsilon}{d\alpha} S_t l_t \frac{1}{2}\rho V^2 =$$
$$= -\frac{\partial C_{Lt}}{\partial \eta}\eta S_t l_t \frac{1}{2}\rho V^2 \qquad (2)$$

where:

- a = incremental (with respect to the initial steady value) wing angle of attack
- η = incremental (with respect to the initial steady value) elevator deflection
- W = aircraft weight
- S = wing surface
- c = wing reference chord
- S_t = surface of the horizontal tail
- l_t = tail arm
- C_L = wing lift coefficient
- C_{Lt} = tail lift coefficient
- C_m = moment coefficient about Y
- J_y = aircraft moment of inertia about Y
- a_t = tail angle of attack
- w = incremental velocity component along Z
- q = angular velocity about Y
- V = airspeed
- ρ = air density

Equations (1) and (2) have been established under the following assumptions, usual in most cases:

(a) the forward speed of the aircraft has been considered constant throughout the manoeuvre;
(b) the component of the aircraft weight along the normal to the flight path has been considered constant;
(c) the tail lift contribution due to the elevator deflection has been neglected in eq. (1);
(d) the tail pitching moment about its aerodynamic center has been neglected in eq. (2);
(e) the effect of the elastic deformation of the aircraft structure has been neglected;
(f) the unsteady flow effects have been disregarded, except for the delay in the downwash appearing at the tail, which is accounted for in the usual way by the fourth term of eq. (2);
(g) the aerodynamic derivatives have been considered constant within the ranges of the effective angles of attack and elevator deflections.

Eqs. (1) and (2) are usually written in non-dimensional form:

$$\frac{d\alpha}{d\tau} + \frac{1}{2} a\,\alpha - \bar{q} = 0 \tag{3}$$

$$\chi \frac{d\alpha}{d\tau} + \omega\alpha + \frac{d\bar{q}}{d\tau} + \nu\bar{q} = -\delta\eta \tag{4}$$

where:

τ = aerodynamic time = $t/\bar{t}$, $(\bar{t} = W/g\rho SV)$

α = w/V

$\bar{q} = \bar{t}\, q$

$a = \frac{\partial C_L}{\partial \alpha}$ (per radian)

X, w, ν, δ are non dimensional derivatives, defined as follows[1]:

$$\chi = -\frac{m_{\dot w}}{i_B}, \quad i_B = \frac{J_y}{(W/g)\, l_t^2}, \quad m_{\dot w} = \frac{\partial M/\partial \dot w}{\rho S l_t^2}, \quad \dot w = \frac{d(w/V)}{dt} = \frac{d\alpha}{dt}$$

$$\omega = -\mu_1 \frac{m_w}{i_B}, \quad \mu_1 = \frac{W/g}{\rho S l_t}, \quad m_w = \frac{\partial M/\partial w}{\rho V S l_t} \tag{5, 6}$$

$$\nu = -\frac{m_q}{i_B}, \quad m_q = \frac{\partial M/\partial q}{\rho V S l_t^2} \tag{7}$$

$$\delta = -\mu_1 \frac{m_\eta}{i_B}, \quad m_\eta = \frac{\partial M/\partial \eta}{\rho V^2 S l_t} \tag{8}$$

If eqs. (3) and (4) are solved for a, a single equation is obtained:

$$\frac{d^2\alpha}{d\tau^2} + 2R\frac{d\alpha}{d\tau} + (R^2+J^2)\,\alpha = -\delta\eta \tag{9}$$

where:

$$R = \frac{1}{2}\left(\nu + \chi + \frac{1}{2}a\right) \tag{10}$$

$$J = \left(\omega + \frac{1}{2}\nu a - R^2\right)^{\frac{1}{2}} \tag{11}$$

[1] The British notation has been used on writing equations (3) and (4). The U.S. equivalent of the British derivatives is given by the following expressions:

$$C_{m d\alpha} = \frac{1}{\mu_1}\frac{2 l_t}{c} m_{\dot w}, \qquad C_{m\alpha} = \frac{2 l_t}{c} m_w$$

$$C_{m d\theta} = \frac{1}{\mu_1}\frac{2 l_t}{c} m_q, \qquad C_{m\delta} = \frac{2 l_t}{c} m_\eta$$

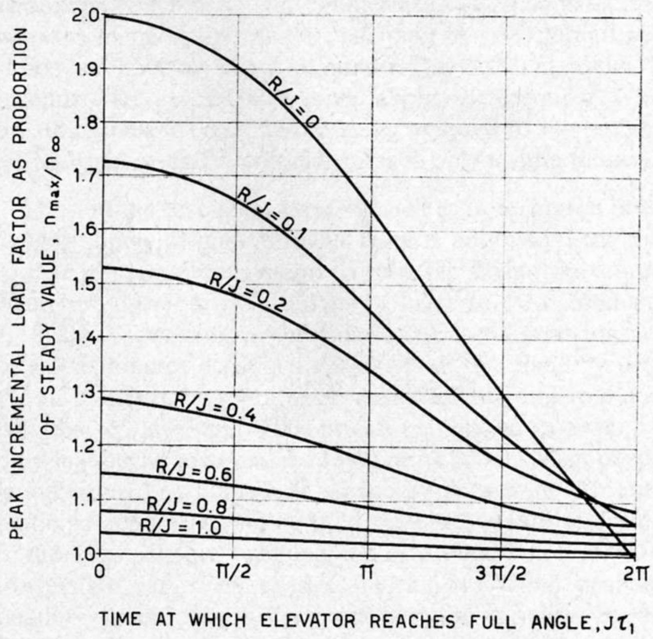


Fig. 1

Eq. (9) is a second order differential equation with constant coefficients, defining the motion of an equivalent one-degree-of-freedom mechanical system. It is easy to recognize that R is the damping coefficient, J is the natural damped frequency and (R^2+J^2) is the natural undamped frequency of the resulting motion. Of course, only the short period oscillation is considered here, as the phugoid motion is

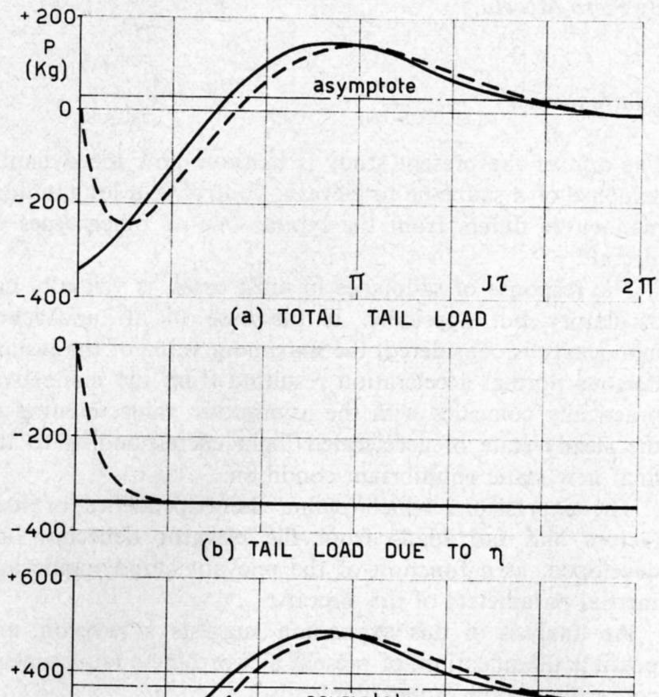


Fig. 2

automatically disregarded, according to assumption (a) (see above).

The solution and discussion of eq. (9) may be found in the literature, where the expressions for the peak normal acceleration (or load factor), the asymptotic normal acceleration, tail angle of attack and tail load are given for different shapes of the disturbing function, $\eta(\tau)$ (see ref. [1], [2] and [3]).

The disturbing functions considered by different authors correspond to checked or unchecked manoeuvres, following different laws of application of the elevator (with linear or exponential or sine-wave portions).

In any case, if unchecked manoeuvres are considered, the most severe conditions, as far as load factors and tail loads are concerned, correspond to the assumption of an abrupt elevator deflection η_0, applied at $\tau = O$ and then maintained indefinitely. Our attention will be limited to this case.

It is obvious that, in such a case, the maximum tail download (for pull-up manoeuvre) develops at $\tau = O$. It is easily understood, moreover, that, with any other law of deflection of the elevator the absolute value of the tail download is lower.

Elevator deflected instantaneously to η_0 and then held.

With initial conditions $\tau = a = q = O$, the solution of eq. (9) can be expressed as (see ref. [1]):

$$\alpha = -\frac{K}{J^2}\,\delta\,\eta_0 \tag{12}$$

where J is given by (11) and

$$K = \frac{1}{(R/J)^2+1}\left[1 - e^{-\frac{R}{J}J\tau}\left(\cos J\tau + \frac{R}{J}\sin J\tau\right)\right] \tag{13}$$

The incremental load factor at C. G. of aircraft is proportional to a and results:

$$n = \frac{a}{C_{LE}}\,\alpha = -\frac{a}{C_{LE}}\,\frac{K}{J^2}\,\delta\,\eta_0 = -\frac{\frac{1}{2}\rho V^2}{W/S}\,a\,\frac{K}{J^2}\,\delta\,\eta_0 \tag{14}$$

where C_{LE} is the lift coefficient of the aircraft at the initial steady condition of flight.

In (12) and (14) only K is variable with time, according to (13). In general, according to this function, n (or a) increases right from the beginning of the manoeuvre, reaches a first maximum n_{max} and then, through decaying oscillations, tends to an asymptotic value n_∞.

According to (13), K reaches ist first maximum K_{max} for $J\tau = \pi$:

$$K_{max} = \frac{1}{(R/J)^2+1}\left(1 + e^{-\pi\frac{R}{J}}\right) \tag{15}$$

It is, therefore, (from (14)):

$$n_{max} = -\frac{a}{C_{LE}}\,\frac{K_{max}}{J^2}\,\delta\,\eta_0 = -\frac{a\delta}{C_{LE}}\,\frac{1+e^{-\pi\frac{R}{J}}}{R^2+J^2}\,\eta_0 \tag{16}$$

The asymptotic value is reached for $J\tau = \infty$:

$$K_\infty = \frac{1}{(R/J)^2+1} \tag{17}$$

It is, therefore:

$$n_\infty = -\frac{a}{C_{LE}}\,\frac{K_\infty}{J^2}\,\delta\,\eta_0 = -\frac{a\delta}{C_{LE}(R^2+J^2)}\,\eta_0 \tag{18}$$

The ratio of (16) to (18) yields:

$$\frac{n_{max}}{n_\infty} = 1 + e^{-\pi\frac{R}{J}} \tag{19}$$

Fig. 1, taken from ref. 4, shows n_{max}/n_∞ for different ratios R/J. The unchecked type of manoeuvre is considered, with an initial linear variation of the elevator deflection up to a value η_0 which is then maintained indefinitely. Time at which full elevator deflection, η_0, is attained, is denoted as $J\tau_1$ (non-dimensional) and plotted as abscissae on fig, 1. Values of n_{max}/n_∞ relating to our particular case of abrupt elevator deflection, correspond to $J\,\tau_1 = O$ on fig.1.

The influence of the ratio R/J on the amplitude of the oscillating motion is evident. R/J is the ratio between the damping coefficient (R) and the natural damped frequency (J) (see (10) and (11)). For R/J tending to zero, n_{max}/n_∞ tends to 2, i. e. the peak incremental normal acceleration, $n_{max}g$, tends to twice the asymptotic incremental normal acceleration, $n_\infty g$. As R/J increases, n_{max}/n_∞ decreases, tending to unity for R/J tending to ∞, i. e. the peak of normal acceleration tends to disappear.

The elevator deflection η_0 leading to n_∞ or n_{max} is immediately derived from (18) or (16):

$$(\eta_0)_{n_\infty} = -\frac{C_{LE}(R^2+J^2)}{a\delta}\,n_\infty \tag{20}$$

$$(\eta_0)_{n_{max}} = -\frac{C_{LE}(R^2+J^2)}{a\delta\left(1+e^{-\pi R/J}\right)}\,n_{max} = \frac{(\eta_0)_{n_\infty}}{1+e^{-\pi R/J}}\,n_{max} \tag{21}$$

Tail load

The general expression for the incremental tail load due to the manoeuvre is:

$$P = \frac{1}{2}\rho V^2 k_t S_t \left(a_t \alpha_t + a_2 \eta\right) \tag{22}$$

where:

S_t = horizontal tail surface

k_t = tail efficiency (dynamic pressure at the tail $= k_t \frac{1}{2}\rho V^2$)

a_t = $\partial C_{Lt}/\partial \alpha_t$ (per radian)

a_2 = $\partial C_{Lt}/\partial \eta$ where C_{Lt} is the tail lift coefficient

a_t = incremental tailplane angle of attack (measured with respect to the tailplane chord)

The general expression for a_t is

$$\alpha_t = \left(1 - \frac{d\varepsilon}{d\alpha}\right)\alpha + \frac{l_t}{V}\,q + \frac{l_t}{V}\,\frac{d\varepsilon}{d\alpha}\,\frac{d\alpha}{dt} \tag{23}$$

The first term of this expression represents the increase in the tail angle of attack due to the increase of the wing angle of attack (with the *static* downwash correction); the second

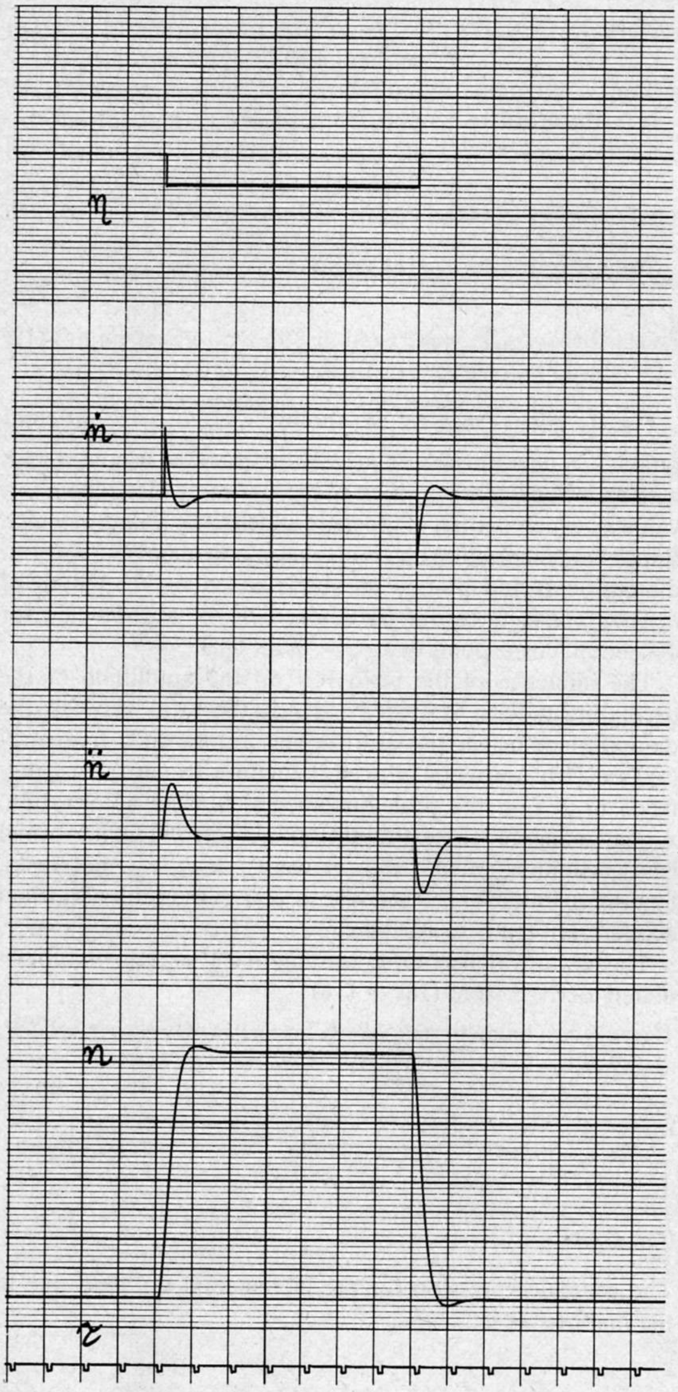


Fig. 3. Response to instantaneous elevator deflections

term represents the increment due to the angular velocity in pitch, q, of the aircraft; the last term accounts for the delay in the downwash appearing at the tail.

By expressing a_t, a and q as functions of the aerodynamic time $\tau = t/\bar{t} = t/(W/g\rho SV)$ eq. (23) becomes

$$\alpha_t = \left(1 - \frac{d\varepsilon}{d\alpha}\right)\alpha + \frac{l_t}{V}\bar{q}\frac{1}{\bar{t}} + \frac{l_t}{V}\frac{d\varepsilon}{d\alpha}\frac{1}{\bar{t}}\frac{d\alpha}{d\tau} \tag{24}$$

If, from eq. (3),

$$\bar{q} = \frac{d\alpha}{d\tau} + \frac{1}{2}a\alpha$$

is introduced in (24), we obtain:

$$\alpha_t = \left(1 - \frac{d\varepsilon}{d\alpha} + \frac{a}{2\mu_1}\right)\alpha + \left(1 + \frac{d\varepsilon}{d\alpha}\right)\frac{1}{\mu_1}\frac{d\alpha}{d\tau} \tag{25}$$

where:

$$\mu_1 = W/\rho g S l_t.$$

The tail load can be written, therefore, (eq. 22):

$$P = \frac{1}{2}\rho V^2 k_t S_t\left(B\alpha + C\frac{d\alpha}{d\tau} + a_2\eta\right) \tag{26}$$

where:

$$B = \left(1 - \frac{d\varepsilon}{d\alpha} + \frac{a}{2\mu_1}\right)a_t$$

$$C = \left(1 + \frac{d\varepsilon}{d\alpha}\right)\frac{a_t}{\mu_1}$$

In the particular case of the abrupt elevator deflection ($\eta = \eta_0 =$ constant), a is given by eq. (12) and the tail loads can be correspondingly determined, as a function of the aerodynamic time, τ, through eq. (26) which becomes:

$$P = \frac{1}{2}\rho V^2 k_t S_t \frac{\delta}{J^2}\left(-BK - CJL + J^2\frac{a_2}{\delta}\right)\eta_0 \tag{27}$$

where K is given by (13) and

$$L = e^{-\frac{R}{J}J\tau}\sin J\tau \tag{28}$$

A typical variation of P with time is shown in fig. 2a, where the full line is for instantaneous elevator deflection and the dotted line is for an exponential law of elevator deflection.

At the beginning of the manoeuvre ($\tau = 0$), when $\eta = \eta_0$ and $K = L = 0$, eq. (27) yields:

$$P_0 = \frac{1}{2}\rho V^2 k_t S_t a_2 \eta_0 \tag{29}$$

$$(\tau = \infty,\ K = J^2/(R^2+J^2),\ L = 0)$$

If a pull-up manoeuvre is considered, P_0 is the maximum incremental download at the tail. The incremental download decreases, in absolute value, as time increases, and may then become positive (upload), as shown in fig. 2a. A maximum upload is reached and then, through a decaying oscillation, the asymptotic value

$$P_\infty = \frac{1}{2}\rho V^2 k_t S_t\left(-B\frac{\delta}{R^2+J^2} + a_2\right)\eta_0 \tag{30}$$

tends to be established.

It is interesting to relate the tail maximum incremental download to the load factor, n_∞ or n_{max}. By introducing in (29) the value (20) of η_0, the following expression is obtained:

$$P_0 = -k_t S_t \frac{W}{S}\frac{a_2}{a}\frac{R^2+J^2}{\delta}n_\infty \tag{31}$$

and, if the expression (11) of J is remembered:

$$P_0 = -k_t S_t \frac{W}{S}\frac{a_2}{a}\frac{1}{\delta}\left(\omega + \frac{1}{2}\nu a\right)n_\infty \tag{32}$$

This expression shows the relevant fact, that the maximum tail incremental download is *independent of the airspeed* and directly proportional to the wing loading. Its value (per n_∞) is determined, moreover, by k_t, S_t, a_2/a and by the aerodynamic derivatives δ, ω, ν and a relating to the particular aircraft considered.

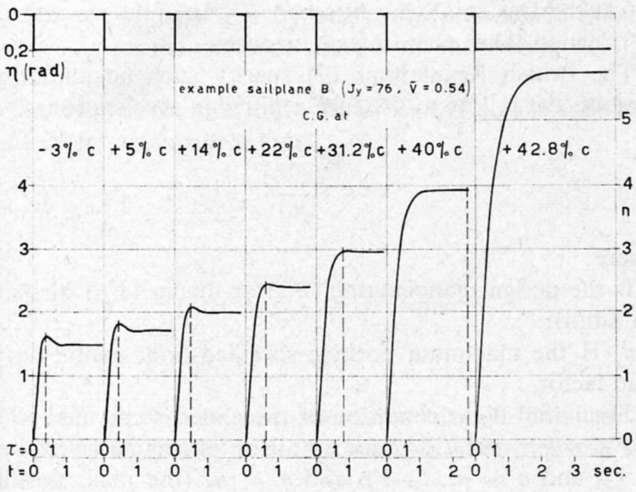


Fig. 4

3 Application to sailplanes

Calculations carried out for two modern sailplanes (a 'standard' single-seater and a heavy two-seater) have shown that the ratio R/J (damping factor to natural frequency of the short period pitching oscillation) *is considerably higher* than on other types of aircraft. The data relating to the two example sailplanes and the corresponding calculated derivatives and coefficients are shown in table I.

In order to investigate the effect of variations of the main parameters involved (C.G. location, moment of inertia J_y, tail volume coefficient $\overline{V}=S_t\ l_t\ /Sc$), the following cases have been considered, relating to example sailplane B (see table I and II, fig. 4, 5, 6):

(1) for the basic $\overline{V} = 0{,}54$ and $J_y = 76$ kgm. sec², *the C. G. location* has been assumed to vary between $-3\%c$ and $+ 42.8\%c$.

(2) for C. G. at $25\%c$ and $J_y = 76$ kgm.sec², *the tail volume coefficient* has been assumed to vary from — 50% to + 50% of the basic value.

(3) for C. G. at 25% c and $\overline{V} = 0{,}54$, *the moment of inertia* has been assumed to vary between — 50% to + 50% of the basic value.

It can be seen from table I and II that, in all these cases, R/J is higher than one, or very close to one, whereas it is known that on other types of aircraft it may, and does usually, assume intermediate values between 0 and 1, often considerably lower than 1.

This fact can be connected to a practical result found by the author (see ref. (6)), relating to the longitudinal dynamic behaviour of a sailplane. It was found (ref. (6)) that the frequency of the so-called 'short period oscillation' is remarkably low, for the example sailplane considered, and of the same order of magnitude as the 'phugoid oscillation'. The ratio between the two frequencies (order of magnitude: 1.5 ÷ 3) is considerably different from the usual ratios pertinent to other types of aircraft (order of magnitude 10; see, for instance, ref. (5)). The consequent low value of J, in the case of sailplanes, may explain the high values of R/J.

It can be seen from eq. (19) that, with R/J values of about 1, or higher, the ratio n_{max}/n_{∞} is $\simeq 1$, i. e, the peak normal acceleration is very close to the asymptotic value.

This fact, together with the heavy damping of the short period oscillation (see, for instance, ref. (6)) clearly explains the typical n,τ response curves of fig. 3 to 6, where the dynamic overshoot of n is seen to be very small, or practically nonexistent.

For obtaining the response curves illustrated in fig. 3, 4, 5, 6, in order to avoid the lengthy calculations involved, use has been made of an analogue recording computer. A typical record is shown in fig. 3.

All curves are for example sailplane B, supposed at an initial condition of steady flight ($C_{LE} = 0{,}257$) and then subjected to an instantaneous elevator deflection of $\eta_o = 0{,}2$ rad $\simeq 11°.5$.

Fig. 4 shows the effect of the C. G. location. Curves of n response are shown for C. G. positions ranging from — 3% to + 42,8%c. It can be clearly seen how the dynamic overshoot n_{max} is considerable for very extreme (and quite unrealistic) forward C. G. locations. As the C. G. is shifted backwards, the peak load factor tends to disappear: the response tends to be aperiodic. More exactly, three intervals of C. G. locations can be distinguished along the wing reference chord, limited by two characteristic points: the C. G. location x_{Jo} corresponding to J = O (R/J = ∞), and

Fig. 5

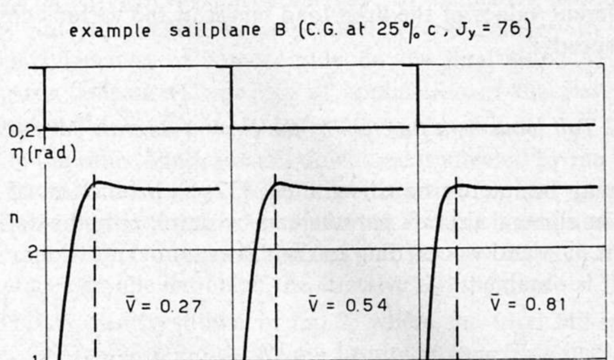
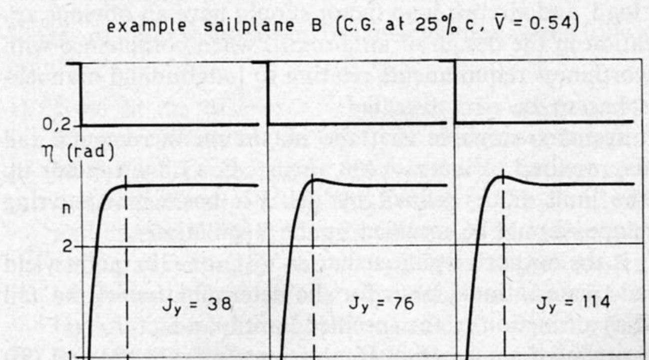


Fig. 6



the stick-fixed manoeuvre point, N_m. In front of x_{Jo} (in our case: $x_{Jo} \simeq 42\%c$), the response is oscillatory. Between x_{Jo} and N_m (in our case: $N_m \simeq 65\%c$), the response is aperiodic. Aft of N_m, the response is divergent. This last case is of no practical interest, N_m being always well aft the stick-fixed neutral point. The asymptotic value n_∞, corresponding to the assumed constant elevator deflection, increases as the C. G. is shifted backwards: this is clear from eq. (18), where the C. G. location is contained in J, which decreases as the C. G. is displaced backwards.

Fig. 5 shows the effect of $\overline{V}$ on the response, in the purely hypothetical case that all other parameters remain unchanged.

Even the important assumed variations of $\overline{V}$ ($\pm$ 50% of the basic intermediate value) practically do not affect the n response, an increase of $\overline{V}$ tending only slightly to accentuate the tendency to an aperiodic response.

Fig. 6 shows the effect of J_y, in the purely hypothetical case that all other parameters remain unchanged. Even the important assumed variations of J_y ($\pm$ 50% of the basic intermediate value) do not affect sensibly the n response, a decrease of J_y tending to accentuate the tendency to an aperiodic response.

It seems worth remarking that variations of J_y lead, in any case, to the same n_∞ (see fig. 6). In fact, from eq. (18) and developing δ, R and J according to their respective expressions, it can be seen that the ratio $\delta/(R^2+J^2)$ is independent of J_y. The same happens with variations of $\overline{V}$ for the particular case considered in fig. 5, the C. G. being located at the neutral point of the tailless aircraft (0.25c). With other C. G. locations however it does not, since ($R^2 + J^2$) is no longer directly proportional to $\overline{V}$, whereas δ always is.

These results suggest the approximation of relating the elevator deflection (and consequent tail load) directly to n_∞ (as taken from the basic *V-n* manoeuvre envelope) rather than to n_{max}. Owing to the small difference, if any, between n_{max} and n_∞, the consequent error involved in the determination of the corresponding elevator deflection or tail load is also small.

3.1 *Airworthiness requirements on manoeuvring tail loads*

The relation between elevator deflection, or consequent tail load, and aircraft load factor should have an obvious application in the design of an aircraft, when compliance with airworthiness requirements relating to longitudinal manoeuvres has to be demonstrated.

It seems reasonable that the maximum incremental tail loads, required to increase the aircraft C. G. load factor up to the limit value defined by the *V-n* basic manoeuvring envelope, should be specified by the Regulations.

It is the author's opinion that eq. (31) or (32) might yield an adequate rational basis for the determination of the tail load as a function of the specified limit load factor.

A survey of some actual Regulations (ref. (7), (8) and (9)) shows, however, that the manoeuvring tail loads arise from specifications which do not, or do not adequately, take into account their dependence on the limit load factor.

The OSTIV (8) and French (9) Regulations specify that:

1) at the Design Manoeuvring Airspeed, V_M, full elevator deflection takes place instantaneously;
2) at the Design Diving Airspeed, V_D, $^1/_3$ of full elevator deflection takes place instantaneously.

The British Regulations (7) specify such an additional load on the tail as to produce a pitching acceleration given by:

$$81.5\,\frac{(n_1-1)^2}{V}$$

where:

V is the design manoeuvring (V_A) or diving (V_D) airspeed (in km/h);

n_1 is the maximum positive specified limit manoeuvring load factor.

The initial flight condition of the glider is specified to be at V_A or V_D, and $n = 1$, for a nose up pitching acceleration; at V_A and $n = n_1$, or V_D and $n = n_2$ (the max. positive specified limit manoeuvring load factor at V_D), for a nose down pitching acceleration.

It is evident that the OSTIV or French Regulations may lead to tail loads capable of producing limit load factors higher that the maximum specified.

This is particularly likely to occur in particular cases. On a two-seater sailplane, for instance, the tail size is determined in most cases, by the necessity of ensuring satisfactory static stability and equilibrium at the different C. G. locations. On this type of glider, the range of the C. G., corresponding to various possible loading conditions (ranging from one single light pilot accommodated in the cockpit, or 'two heavy pilots plus equipment'), is particularly wide. The consequent large tailplane and elevator may well lead to very high tail loads, if the OSTIV or French specifications are applied. Such high loads may be quite unlikely to occur in real flight conditions, owing to the high C. G. normal acceleration that would consequently be attained, corresponding to higher limit load factors than specified.

The British Regulations show an attempt to relate the incremental tail load to the incremental load factor and the airspeed. It is not known to the author, however, on which basis the pitching acceleration (and, therefore, the tail load) is related to $(n_1 - 1)^2/V$.

Eq. (31) shows that, within the limits of the underlying assumptions, P_o is directly proportional to the incremental normal load factor, n_∞, and independent of the airspeed.

The only dependence on the airspeed derives indirectly from the fact that the regulations may, and usually do, specify different values of the limit load factor at the various design airspeeds.

3. 2 *Tail loads as a function of the current aircraft parameters*

It may be interesting to write eq. (32) as a function of the main current aircraft parameters, by developing the derivatives δ, ω and v according to their common expressions.

It is obtained:

$$P_o = -S_t\,\frac{W}{S}\,\frac{1}{\overline{V}}\left\{N_{tailless} - x_{cG} + a_t\,\overline{V}\,k_t\left[\frac{1}{a}\left(1-\frac{d\varepsilon}{d\alpha}\right)+\frac{\rho g l_t}{2W/S}\right]\right\}(n_{lim}-1) \tag{33}$$

where:

$\overline{V}$ = tail volume coefficient

$N_{tailless}$ = neutral point of the aircraft less tail (expressed as a fraction of the reference chord from the leading edge);

x_{CG} = center of gravity location (expressed as a fraction of the reference chord from the leading edge);

$n_{lim} - 1 = n_\infty$.

The derivatives δ, ω and ν have been given the following usual theoretical expressions:

$$\delta = -\mu_1 \frac{m_\eta}{i_B} = \frac{(W/g)^2 c}{2 \rho S J_y} k_t \bar{V} a_2 \tag{34}$$

$$\omega = -\mu_1 \frac{m_w}{i_B} = \frac{(W/g)^2 c}{2 \rho S J_y} \left[a (x_{cg} - N_{tailless}) - k_t a_t \bar{V} \left(1 - \frac{d\varepsilon}{d\alpha}\right) \right] \tag{35}$$

$$\nu = -\frac{m_q}{i_B} = \frac{(W/g)^2 c}{2 \rho S J_y} k_t a_t \bar{V} \frac{\rho g l_t}{W/S} \tag{36}$$

Eq. (33) shows an influence of the altitude (through air density, ρ) on the tail load: the max tail load occurs at low altitude.

For a given aircraft at a given total weight (W), P_o is affected by the C. G. location: tail load increases (linearly) as the C. G. is shifted forwards. At a given C. G. location, P_o increases with W.

P_o is independent of the moment of inertia of the aircraft: this is obvious from the fact that an instantaneous elevator deflection has been assumed.

The influence of variations of important design parameters, as $\bar{V}$, S_t, a_t, l_t, can be easily studied from (33).

For the sake of an eventual airworthiness specification on tail manoeuvring loads, eq. (33) may be simplified by introducing conservative values of some parameters. Thus, with:

$$k_t = 1 \qquad N_{tailless} = 0.25 \qquad \rho = 0.125 \text{ kg.m}^{-4}\text{.sec.}^2 \qquad g = 9.81 \text{ m.sec}^{-2}$$

eq. (33) becomes:

$$P_o = -S_t \frac{W}{S} \frac{1}{\bar{V}} \left[0.25 - x_{cg} + a_t \bar{V} \left(\frac{1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + 0.613 \frac{l_t}{W/S} \right) \right] (n_{lim} - 1) \tag{37}$$

where P_o, l_t, S_t and W/S are expressed in kilograms, meters, sq. meters and kg./sq.m., respectively.

3. 2 *Additional remarks*

The assumption of abrupt elevator deflection leads to conservative values of the tail download.

In general, this assumption is also conservative with respect to n_{max}. In the particular case of sailplanes, however, it is not so: in fact, n_{max} practically coinciding with n_∞ (which depends, of course, only on the final value of the elevator deflection), any law of application of the elevator would lead (practically) to the same n_∞ ($\simeq n_{max}$).

On the other hand, the tail download is affected by the law of elevator deflection. When the deflection is not instantaneous, at the time the maximum deflection is attained the aircraft will have assumed an angular velocity in pitch, that produces an alleviation of the effective tail incidence.

This is clearly shown in fig. 2, where the total tail load (fig. 2a), corresponding to an instantaneous (full line) or exponential (dotted line) law of elevator deflection, is broken into two components (see eq. (22)):

1) the incremental download, due to the incremental elevator deflection, η (fig. 2b);
2) the incremental upload due to the incremental tailplane incidence, a_t (fig. 2c).

In the case of instantaneous elevator deflection, when $t = O$, $\eta = \eta_0$ and $a_t = O$. The incremental tail load, therefore, consists simply of the incremental download due to η.

In the case of a gradual manoeuvre, at the time the maximum elevator deflection is attained: $\eta = \eta_0$ and $a_t \neq O$. The incremental download, therefore, is alleviated by the corresponding upload.

Only the unchecked type of manoeuvre has been considered in this study. The checked type of manoeuvre is such that, a short time after full elevator deflection is reached, the elevator angle is reversed down to a lower angle, which may or may not be the initial one. If the law of application of the elevator in the first part of the manoeuvre is the same as in the unchecked manoeuvre, it is evident that the tail download will also be the same. On the other hand, the C. G. max. normal acceleration cannot be higher than with the unchecked manoeuvre. Such a type of unchecked manoeuvre, therefore, has no interest from our point of view.

Another type of checked manoeuvre, however, can be envisaged, such that the elevator is deflected beyond the angle of the unchecked manoeuvre and immediately reversed to an angle that will give the same limiting normal acceleration. It is evident that such a manoeuvre may lead to higher tail download than the unchecked one.

It seems difficult to cover also this case by a simple airworthiness requirement. In any case, it is interesting to consider that, as stated in par. 2.1, n_{max} is reached after $J\tau = \pi$, i. e. $\tau = \pi/J$ and $t_{nmax} = E\pi/J = (W/S/\rho g V)\ (\pi/J)$ *sec.*, in the case of instantaneous elevator deflection.

For the two example sailplanes A and B, it is:

$$t_{n_{max}} = 0.61 \text{ sec.}, \qquad t_{n_{max}} = 0.85 \text{ sec.},$$

respectively.

These are short times: any relieving manoeuvre following the rapid increment of elevator angle should be made quickly enough, if the normal acceleration must be prevented from exceeding its limiting value.

It is the author's opinion that the simple assumption of instantaneous unchecked manoeuvre is conservative enough in any case, as far as the determination of tail loads on sailplanes for design purposes is concerned.

This opinion is supported by consideration of the following points:

1) Even in the case of a very sharp displacement of the control column by the pilot, the elevator deflection will follow with some delay (and probably according to a quasi-exponential law.) This is due to the aerodynamic and inertial elevator hinge moments to be overcome, and to the stretching of the control transmission under the combined effects of aerodynamic and inertial loads (arising from the whole longitudinal control system).
2) Aeroelastic effects, mainly due to the flexural deformation of the fuselage, produce an alleviation of the tail load calculated on the assumption of infinite rigidity. Calculations (at present unpublished, but mentioned in ref. (4)) taking into account static or dynamic aeroelastic effects, have been carried out by Czaykowsky for particular powered aircraft.

3) The theory discussed here is based on the assumption of steady lift coefficients on the wing and on the tail. It is well known that, with rapid variations of the angle of attack, there is some delay before the lift coefficients attain the values corresponding to the steady state.

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Table I

				Sailplane A	Sailplane B
Wing span	b	=	m	15	18.15
Wing surface	S	=	m²	13.1	17.4
Wing aspect ratio	A	=		17.1	19
Total weight	W	=	kg	300	570
Wing loading	W/S	=	kg/m²	23	32.8
Wing reference chord = m.a.c.	c	=	m	0.94	1.06
Wing lift curve slope	a	=	rad⁻¹	5.39	5.42
Horiz. tail surface	S_t	=	m²	1.6	2.48
Tail arm	l_t	=	m	3.7	4
Tail volume coefficient	V	=		0.485	0.54
Tail lift curve slope	a_t	=	rad⁻¹	4.3	4.1
Tail efficiency	k_t	=		1	1
Downwash factor	$1-\frac{d\varepsilon}{d\alpha}$	=		0.75	0.75
	a_2	=	rad⁻¹	2.71	2.42
Moment of inertia about Y	J_y	=	kgm.sec²	16.2	76
Design manoeuvring speed	V_A	=	m/sec	37.1	45
Limit load factor at V_A	n_1	=		5.3	5.3
Lift coefficient at V_A, n=1	C_{LE}	=		0.267	0.257

Table II

			Sailplane A	Sailplane B
Aerodynamic derivatives, for	$\overline{V}$	=	0.485	0.54
	J_y	=	16.2	76
	ω	=	19.85	12.85
	χ	=	1.75	0.9
	ν	=	7.75	3.97
	δ	=	22.15	14.15
'damping coefficient'	R	=	6.1	3.79
'natural damped frequency'	J	=	1.88	3.03
	R/J	=	*3.24*	*1.25*
Effect of variations of CG, $\overline{V}$ and J_y:				
for $\overline{V}$ = 0.27, (J_y = 76, CG at 25% c):	R	=		2.57
	J	=		2.74
	R/J	=		*0.94*
for $\overline{V}$ = 0.81, (J_y = 76, CG at 25% c):	R	=		5
	J	=		3.24
	R/J	=		*1.54*
for J_y = 38, ($\overline{V}$ = 0.54, CG at 25% c):	R	=		6.22
	J	=		2.91
	R/J	=		*2,14*
for J_y = 114, ($\overline{V}$ = 0.54, CG at 25% c):	R	=		2.97
	J	=		2.62
	R/J	=		*1.13*
for CG at 14% c, ($\overline{V}$ = 0.54, J_y = 76):	R	=		3.79
	J	=		3.95
	R/J	=		*0.96*
for CG at 40% c, ($\overline{V}$ = 0.54, J_y = 76):	R	=		3.79
	J	=		0.63
	R/J	=		*6.02*

(Swiss Aero-Revue 5–6/1967)