Influence of Elastic Wing-twisting on certain Characteristics of longitudinal Stability and Controllability of a Glider

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List of Symbols – Bezeichnungen

b	= wing chord – Flügeltiefe
b_{aver}	= average wing chord – mittlere Flügeltiefe
b_b	= elevator chord – Höhenrudertiefe
k_w	= elevator gearing – Höhenruderübersetzung
m_w^a	= $dC_H/dd_{hor.\ tail}$ — where C_H = elevator hinge moment coefficient / wobei C_H = Höhenruder-Momentenbeiwert
m_w^δ	= $d\,C_H/d\delta_b$ — where C_H = elevator hinge moment coefficient / wobei C_H = Höhenruder-Momentenbeiwert
m_z	= pitching moment – Nick- oder Längsmoment
p	= wing loading – Flächenbelastung
$\overline{x_f}$	= aerodynamic centre coefficient – «Druckmittelpunkt-Beiwert»
$\overline{x}_{r.c.}$	= rigidity centre coefficient – «Beiwert der elastischen Achse»
Z	= distance from wing root – Abstand von der Flügelwurzel
δ_b	= elevator angle – Höhenruderwinkel
ξ_K	= wing twist, between tip and root – Flügelschränkung zwischen Spitze und Wurzel
∅	= tail setting to wing – Höhenleitwerkseinstellwinkel zum Flügel

All other symbols have their usual meanings or are defined in the text.
Alle anderen Bezeichnungen haben die übliche Bedeutung oder sind im Text definiert.

Introduction

Wings of modern high-performance gliders feature high aspect ratios ($\lambda = 25$–30), thin skins ($\delta = 0.8$—1.5 mm) and extensively curved laminar profiles. Such wings have low torsional rigidity and are loaded with high torques. This necessitates careful consideration of elastic wing twisting while calculating the longitudinal stability and controllability of the glider.

Methods for such calculations are well developed as far as aeroplane structures are concerned. But they are rather inconvenient for practical calculations for gliders, as some of them are too simplified and can serve only for the qualitative evaluation, while others, though precise, are too complicated and laborious.

The above constructional and aerodynamic features of gliders, as well as some differing characteristics from those of aeroplanes (optimum tapered wings which are almost unswept, lack of sizable cut-outs in wing skins, relatively small fuselage) permit certain assumptions which are not detrimental to precision and at the same time greatly simplify the calculations.

Principal assumptions

1. The wing is composed of absolutely identical profiles.
2. Wing sweep is negligible.
3. Rigidity centres of all wing sections are located along a constant percent line, i. e. $\bar{x}_{r.c.}$ (Z) = constant.
4. The wing is normal to the pitching plane of the glider.
5. Only small deformations are considered.
6. Only linear dependencies are considered. $C_y = f(a)$ and $m_z = f(a)$.
7. The distribution of lift over the span of the wing is considered elliptical.
8. The influence of the drag due to wing twisting on the pitching moment is negligible.
9. The influence of the change in effective fuselage incidence, due to elastic wing twisting, on the pitching moment is negligible.

Relation between wing twist due to elastic distortion and lift coefficient

An integro-differential equation illustrating the dependence between wing tip and wing root section twist angles is of the form

$$\xi_K = q\int_0^l \frac{dz}{GJ_p(z)} \int_l^z \left[C_{m_0\,sect} + \bar{a}\, C_{y\,sect}(z, \xi_K)\right] b^2(z)\, dz \qquad (1)$$

where GI_p (z) = wing torsional rigidity,

$$\bar{a} = \bar{x}_{N.A.} - \bar{x}_f$$

= distance between the rigidity centre (flexural axis) and mean aerodynamic centre of a wing section as fractions of its chord.

At zero lift, the section pitching moment coefficient $C_{mo\ sect.}$ does not depend on the spanwise coordinate z due to assumption 1.

Thus, section lift coefficient can be expressed as

$$C_{y\,sect}(z, \xi_K) = C_{y_0} + \Delta C_{y\,twist}(z, \xi_K) \qquad (2)$$

where C_{yo} (for the untwisted wing) does not depend on z (assumption 7) or ξ_k, and is numerically equal to the lift coefficient for the whole wing. Also $\Delta\ C_{y\ twist} = f(z, \xi_k)$ but does not influence the overall wing lift coefficient.

$$\Delta C_{y\,twist} = \frac{\Gamma_{1z}(z)\, b_{aver} \cdot \xi_k^{\circ}}{b(z)} \tag{3}$$

where $\Gamma_{1z}(z)$ is the additional non-dimensional circulation (related to chord and velocity of the undisturbed flow) produced by wingtip section twist ξ_k of 1°.

Section values of $\Gamma_{1z}(z)$ can be found by the methods Risberg or Multhopp, for any variation of $\frac{\xi(z)}{\xi_k}$ across the span resulting, for instance, from static wing structural tests (assumptions 5 and 6).

By putting dependencies 2 and 3 into equation 1 and substituting

$$A = 57.3 \int_0^{l} \frac{dz}{GI_p(z)} \int_l^{z} b^2(z)\, dz$$

and

$$B = b_{aver} \int_0^{l} \frac{dz}{GI_p(z)} \int_l^{z} \Gamma_{1z}(z)\, b(z)\, dz$$

which after simple conversions are reduced to

$$\xi_k^{\circ} = \frac{q(C_{m_0} + \bar{a} C_y) A}{1 - q \bar{a} B} \tag{4}$$

Equation of moments and trim curve of a glider with elastic wing

While flying at a given C_y the root section incidence of a twisted wing is $\Delta a_0 (\xi_k)$, higher than that of an untwisted wing due to redistribution of the circulation.

At the same time the wing pitching moment, fuselage incidence, geometrical incidence of the horizontal tail, and downwash angle at the tail are subjected to changes.

From assumptions 2, 8 and 9 the determination of changes in the pitching moment of the glider is in practice reduced to the determination of the change in the pitching moment due to the horizontal tail.

As the values of additional circulation for each section are in linear dependence on ξ_k, then $\Delta a_0 = f(\xi_k)$ is a linear function as well. Thus we can say that $\Delta a_0 = \frac{\partial \alpha_0}{\partial \xi_k} \cdot \xi_k$ or after substitution in equation (4)

$$\Delta \alpha_0 = \frac{\partial \alpha_0}{\partial \xi_k} \cdot \frac{q(C_{m_0} + \bar{a} C_y) A}{1 - q \bar{a} B} \tag{5}$$

The value of $\frac{\partial \alpha}{\partial \xi_k}$ can be taken from calculations for additional circulation by wing twisting.

The approximate value of additional downwash at the tail in the case of high aspect ratio wings can be determined without significant error from

$$\Delta \varepsilon = D \cdot C_y^{\alpha} \cdot \Delta \alpha_0 (\xi_k)$$

where coefficient D depends only on the geometry of an ideal rigid glider, the calculations for which are not too difficult.

Then, using equation 5, we come to

$$\Delta \varepsilon = D \cdot C_y^{\alpha} \cdot \frac{\partial \alpha_0}{\partial \xi_k} \cdot \frac{q(C_{m_0} + \bar{a} C_y) A}{1 - q \bar{a} B} \tag{6}$$

The pitching moment of a glider with neutral elevator angle is

$$m_z = m_{z\,wing} - K \cdot A_{hor.\,tail}\, a_{hor.\,tail} (\alpha_{wing} + \phi - \varepsilon)$$

After putting in equations 5 and 6 taking

$$C = -K \cdot A_{hor.\,tail}\, a_{hor.\,tail} \frac{\partial \alpha_0}{\partial \xi_k} (1 - D \cdot C_y^{\alpha})$$

we come to

$$m_z = m_{z\,rigid} + \frac{q(C_{m_0} + \bar{a} C_y) A C}{1 - q \bar{a} B} \tag{7}$$

where m_z rigid is the pitching moment of a glider with an ideal rigid wing.

It should be noted that C is determined only by the geometry of an ideal rigid glider.

Using the dependence

$$\delta_b = - \frac{m_z (\delta_b = 0)}{m_z^{\delta}}$$

we get a twin curve equation for a glider with an elastic wing

$$\delta_b = \delta_{b\,rigid} - \frac{q(C_{m_0} + \bar{a} C_y) A C}{1 - q \bar{a} B} \tag{8}$$

Limit of static longitudinal stability with angle of attack

The value of static longitudinal stability with angle of attack appears as a partial derivative $\frac{\partial m_z}{\partial C_y}$ at $V = \text{const.}$

$$m_z^{C_y} = m_{z\,rigid}^{C_y} + \frac{q \bar{a} A C}{1 - q \bar{a} B} \tag{9}$$

Limit of static longitudinal stability with speed

Generally, the limit of static longitudinal stability with speed is found through

$$\frac{d m_z}{d C_y} = m_z^{C_y} + \frac{\partial m_z}{\partial V} \left(\frac{dV}{d C_y} \right) (M_y = 1)$$

with $m_z\ C_y$ value determined through formula 9. The influence of compressibility can be neglected as all gliders have $V_{max} = 250\text{–}300$ km/per hour. Then, bearing in mind that under steady flight conditions $\frac{dV}{dC_y} = -\frac{V}{2\,C_y}$ and $C_y = p/q$ where p is wing loading we eventually come to

$$\frac{d m_z}{d C_y} = m_{z\,rigid}^{C_y} - \frac{q^2 (C_{m_0} + p \bar{a}^2 B) A C}{p (1 - q \bar{a} B)^2} \tag{10}$$

Elevator control force

Generally

$$P_b = -k_w q S_b b_b (m_w^{\alpha} \cdot \alpha_{hor.\,tail} + m_w^{\delta} \cdot \delta_b)$$

Putting in expressions 5, 6 and 8 we get an equation for steady straight flight

$$P_b = P_{b\,rigid} - k_w q S_b b_b \frac{q^2 (C_{m_0} + \bar{a} C_y)}{1 - q \bar{a} B} \left[m_w^{\alpha} \cdot \frac{\partial \alpha_0}{\partial \xi_k} (1 - D \cdot C_y^{\alpha}) - \frac{m_w^{\delta}}{m_z^{\delta}} \cdot C \right]$$

Comparison between theoretical results and test flight data

To evaluate the degree of approximation in the given formulas experimental data obtained during KAI-19 test flights were used. Fig. 1 presents trim curves calculated theoretically with and without allowance for elastic wing twisting, for centre of gravity $\bar{x}_T = 0.40$, as well as experimentally found points. The horizontal tail angle in relation to the wing root chord is in this case equal to $\varphi = -6^0$. The test points agree well with the curve calculated with allowance for twisting; this fact illustrates satisfactory reliability of the suggested method.

Conclusions and recommendations

After taking the qualitative analysis of the above formulae and bearing in mind that for operational structures

$$C_{m_0} < 0, \quad \bar{\sigma} > 0, \quad \frac{\partial\alpha}{\partial\xi_k} < 0, \quad A > 0, \quad B > 0,$$
$$c > 0, \quad D.C_y^{\alpha} < 1, \quad q\bar{\sigma}B < 1, \text{ and } p.\bar{\sigma}^2 B < |C_{m_0}|$$

the following conclusions can be drawn:

1. Elastic wing twisting results in a decrease in pitching moment and elevator trim angles, the decrease being proportional to q.
2. Values of static longitudinal stability with angle of attack and speed are also decreased, the former by an amount proportional to q and the latter by an amount proportional to q^2). Due to coefficients contained in the formulae the difference in decrease of these values with speed is still more marked. Calculations made for the KAI-19 glider showed that at V = 250 km/per hour the value of static longitudinal stability with angle of attack became decreased by about 0.01, and with speed by about 0.45.

This means that the influence of wing twisting primarily affects the long-period motion, and in the case of low wing rigidity it may lead to periodic or aperiodic instability at high speeds. However, as the long-period motion increases amplitude only very slowly, such instability is not necessarily dangerous.

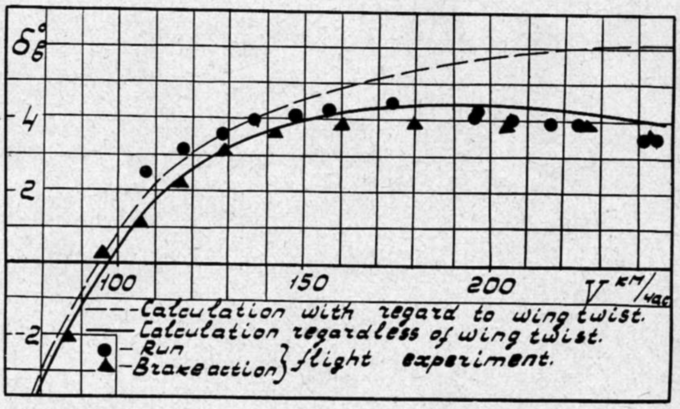
3. Elevator control force under steady flight conditions decreases with increase in speed in proportion to q^2. In the case of a low-rigidity wing it may result in abnormal velocity gradient.

The following recommendations can be made on the basis of the above:

1. Theoretical conclusions as to the longitudinal stability of gliders should be drawn only on the basis of calculations with consideration of elastic wing twisting.
2. Such calculations should be made during the drawing board stage of the design, so that the wing rigidity could if need be increased.
3. One of the main parameters influencing the degree of elastic wing twisting is C_{mo} which depends on the profile curvature. The elastic wing twisting and its undesirable effects can be greatly decreased, if a low-curvature profile is selected for the wing.

The resulting lower flight performance of the glider at low speeds can be regained by mechanically changing the profile.

Fig. 1



Einfluss der elastischen Flügelverdrehung auf gewisse Eigenschaften der Längsstabilität und Steuerbarkeit eines Segelflugzeuges

(Dieser Bericht ist eine gekürzte Übersetzung des russischen Originals)

Moderne Segelflugzeug-Flügel haben hohe Seitenverhältnisse von 25–30 und dünne Flügelbeplankungen von 0,8–1,5 mm Dicke. Sie werden hohen Drehbelastungen unterworfen. Deshalb sollte die elastische Verdrehung bei der Berechnung der Längsstabilität und Steuerbarkeit berücksichtigt werden.

Der Bericht umreisst eine geeignete Methode für Segelflugzeuge mit folgenden vereinfachenden Annahmen:

1. Das Flügelprofil ist konstant über die Spannweite.
2. Die Pfeilung ist vernachlässigbar.
3. Die elastische Achse hat in allen Flügelschnitten die gleiche prozentuale Rücklage.
4. Der Flügel hat keine V-Stellung.
5. Die Deformation ist klein.
6. Auftrieb und Luftkraft-Moment ändern sich linear mit der Verdrehung.
7. Die Auftriebsverteilung ist elliptisch.
8. Das Moment des Luftwiderstandes infolge Verdrehung ist vernachlässigbar.
9. Das Moment der von der Verdrehung herrührenden Änderung des Rumpfwinkels ist vernachlässigbar.

Gleichungen werden angegeben für die Verdrehung zwischen Flügelspitze und -wurzel (Glchg. [1] und [4]) und für den entsprechenden Auftrieb (Glchg. [2] und [3]). Von der Änderung des Nullauftriebswinkels (Glchg. [5]) wird die Änderung des Abwindwinkels am Leitwerk gefunden (Glchg. [6]). Das Nickmoment für das elastische Segelflugzeug ist in Glchg. (7) und der Höhenruder-Trimmwinkel in Glchg. (8) im Verhältnis zu den Werten des entsprechenden starren Segelflugzeuges angegeben. Die partiellen Derivativa des Nickmoments bezogen auf Anstellwinkel und Geschwindigkeit sind in Glchg. (9) und (10) dargestellt. Schliesslich wird noch ein Steuerkraftausdruck in Glchg. (11) gegeben.

Bild 1 zeigt die Trimmkurven für die KAI-19, berechnet mit und ohne elastische Verdrehung, sowie Versuchsergebnisse. Diese stimmen gut mit den Kurven für Verdrehung überein, so dass man die Rechenmethode als zuverlässig bezeichnen kann.

Die Gleichungen zeigen, dass die Stabilität durch Verdrehung vermindert wird: die Anstellwinkel-Derivativa durch einen Betrag proportional zu q und die Geschwindigkeits-Derivativa durch einen Betrag proportional zu q^2. Die entsprechenden Änderungen für die KAI-19 sind 0,01 und 0,45. Die Ergebnisse werden wesentlich beeinflusst durch C_{mo}. Daraus folgt, dass die Verdrehung im Entwurf berücksichtigt werden sollte, und dass es vorteilhaft ist, C_{mo} klein zu halten; der daraus folgende Verlust an Leistungen bei geringer Geschwindigkeit könnte wieder gewonnen werden durch mechanische Änderung des Profils.

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