Computer Calculations on optimum Rate of Climb in parabolic Thermals

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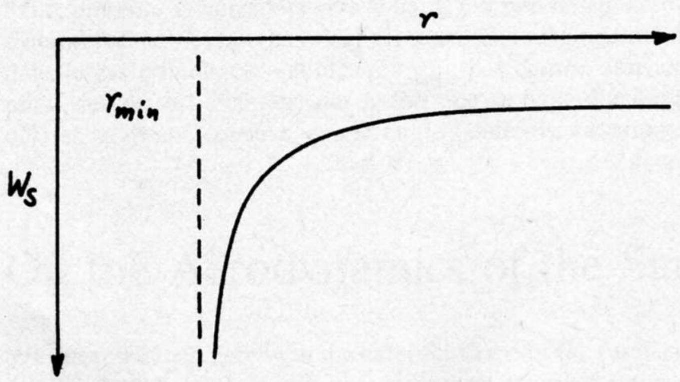
1. Circling Performance

Theoretically there are no difficulties in the calculation of the circling performance of gliders. It is however a tremendous amount of work to allow for the non-constant lift distribution over the radius of the thermal. To cope with the problem an investigation in which all interesting parameters were varied, was made on a digital computer.

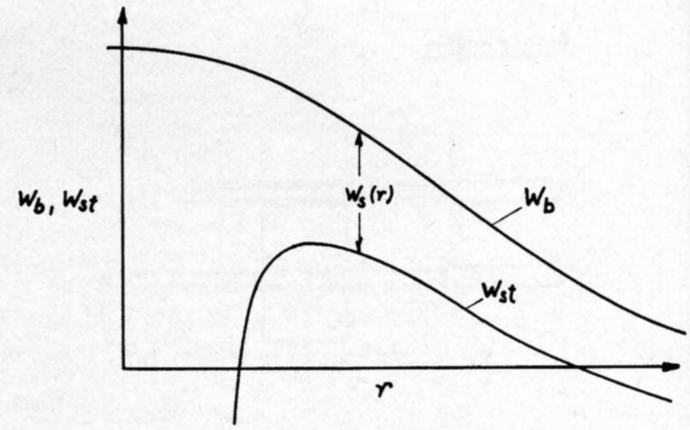
The sinking speed can be calculated by the following formula:

$$(1) \quad w_s = \left(\frac{c_A^2}{\pi \Lambda} \xi + c_{ws} + c_{wp}\right) \cdot c_A^{-3/2} \cdot \sqrt{\frac{2}{\rho} \frac{G}{F}} \cdot \left[1 - \left(\frac{2 \cdot G/F}{\rho \cdot c_A \cdot r \cdot g}\right)^2\right]^{-3/4}$$

The nature of the variation of sinking speed with radius is as shown in the following sketch.



Having given the lift distribution of thermals as shown in figure 1, is it easy to obtain the radius for the optimum upward velocity and the optimum velocity itself, by graphically subtracting the sinking speed from the upward speed of the thermal, as shown in the second sketch.



Analytically the lift distribution will be given by the following parabolic function:

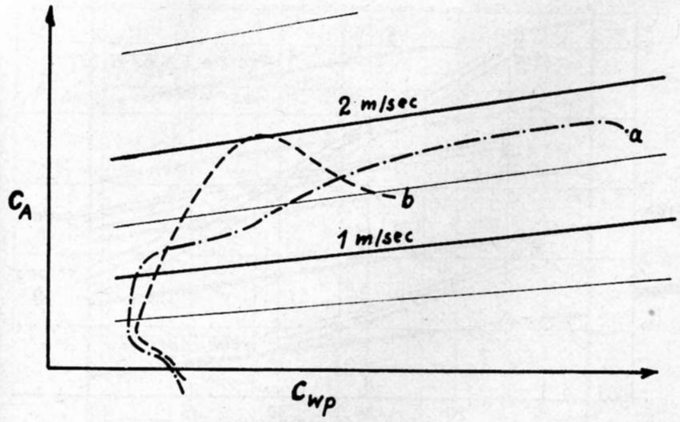
$$(2) \quad W_b = a \cdot r^2 + b \cdot r + c$$

Looking at formula (1) we can find four principal parameters: c_A, c_{wp}, G/F and Λ. For any given thermal lift distribution, we can find the upward velocity (w_{st}) of any glider having a particular set of values. It is supposed that a good pilot is able to find the best angle of bank.

The calculations were made for a wing with elliptic lift distribution

$$\left(c_{wi} = \frac{c_A^2}{\pi \Lambda} \xi \quad \text{with} \quad \xi = 1.0685\right)$$

and a drag coefficient of fuselage and tailplane of c_{ws} = 0,004. Results are given as isoclimbs in diagrams A1 to A23 (eleven are reproduced here. Ed.) of the form shown in the third sketch. If another c_{ws} should be considered the difference must be added to c_{wp} in entering the diagram.



In the sketch, both profile polars have the same maximum lift coefficient but polar b gives better upward velocity:
(a) $w_{st} = 1{,}7$ m/sec.
(b) $w_{st} = 2{,}0$ m/sec.

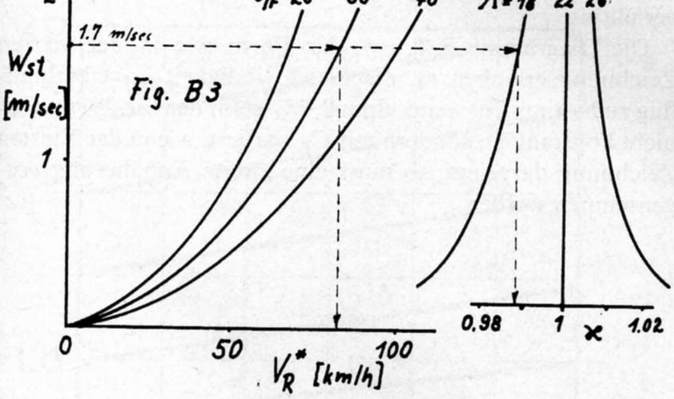
For convenience of working it is useful to draw the profile polar on transparent paper. If area increasing flaps are used, all lift and drag coefficients must be related to the same wing area used for G/F.

Results for the Profile of D-36* (Wortmann 62 K 131) are given in figure 2. The most important parameter is the wing loading, while the influence of aspect ratio is not very great.

2. *Cross country speed*

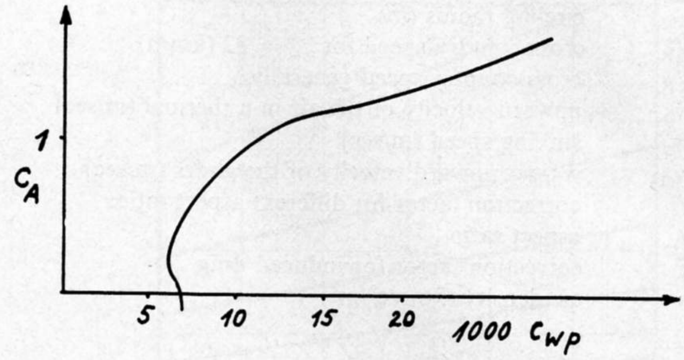
Circling performance is only one part of cross country speed. With the diagrams B1 etc. of the form illustrated in the fourth sketch, we can find the optimum cross country speed V_R in terms of the upward velocity w_{st} (Diagrams A1 etc.) and the parameters G/F, Λ and $c_{wo} = c_{wp} + c_{ws}$ (c_{wp} at low lift coefficients). For example, if $w_{st} = 1{,}7$ (m/sec); $G/F = 30$ (kp/m); $\Lambda = 18$ and $c_{wo} = 0{,}010$, we find that

$$V_R = V_R^* x = 79 \times 0{,}988 = 78 \,(\text{km/h})$$



Possibly the drag c_{wp}, and hence c_{wo}, will depend on c_A as shown in the fifth sketch below. In this case we must notice the lift coefficient for straight-on flying. For example, assume $w_{st} = 0{,}9$ (m/sec); $G/F = 30$ (kp/m); $\Lambda = 22$, and $c_{ws} = 0{,}004$.

A first approximation guess $c_{wp\ min}$ as 0,006 so that $c_{wp\ min} + c_{ws} = 0{,}006 + 0{,}004 = 0{,}010$. We find from diagram B3



that $V_R = V_R^* \times 1 = 56$ km/h and $c_A \approx 0{,}45$. For a second approximation we find from the polar that the real c_{wp} for $c_A = 0{,}45$ is 0,007. Thus our c_{wo} is really 0,011. Because there does not exist a diagram for $c_{wo} = 0{,}011$ we must interpolate between $c_{wo} = 0{,}010$ and $c_{wo} = 0{,}012$ (diagrams B3 and B4) from which we find that $V_R = V_R^* X = 54{,}5$ (km/h); $c_A \approx 0{,}44$.

In most cases it will be sufficient to consider the drag between $c_A = 0{,}15$ and $c_A = 0{,}5$ as onstant.

Putting together the results of circling performance and cross country flying we will find a diagram as shown in figure 3.

3. *Conclusions*

The paper gives a method of determining with the aid of diagrams the cross country speed of a sailplane as a function of wing loading and aspect ratio. Three different lift distributions in thermals are taken into consideration. Only the profile polar need be given.

The method is suitable for comparing different profiles including those with area increasing flaps.

References:

(1) Raspet und Györgyfalvy: «Der Phönix», Zeitschrift für Flugwissenschaften 8 (60), Heft 9.
(2) Eppler: Kreisflug von Segelflugzeugen. Zeitschrift für Flugwissenschaften 1954, Heft 1.
(3) Carmichael: What Price Performance? Soaring 18, May/June 1954.

List of Symbols

a, b, c	factors of parabolic lift distribution in a thermal
c_A	lift coefficient
c_w	drag coefficient
c_{wi}	induced drag coefficient
c_{wp}	profile drag coefficient
c_{ws}	fuselage and tailplane drag coefficient based on wing area
c_{wo}	$c_{wp} + c_{ws}$
F	wing area (m²)
G	weight (kp)
g	acceleration due to gravitation (m/sec²)

* See OSTIV-Publication VII

r	circling radius (m)
V_R^*	cross-country-speed for $\Lambda = 22$ (km/h)
V_R	cross-country-speed generally
w_b	upward velocity of the air in a thermal (m/sec)
w_s	sinking speed (m/sec)
w_{st}	$w_b - w_s$ upward velocity of the glider (m/sec)
$\varkappa$	correction factor for different aspect ratios
Λ	aspect ratio
ξ	correction factor for induced drag
ρ	air density (kp.sec²/m⁴)

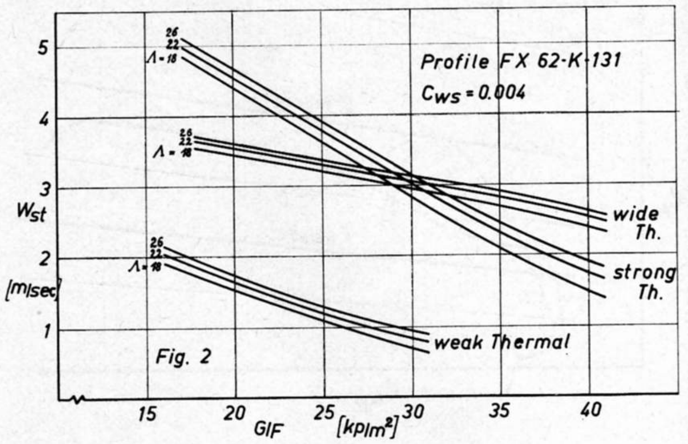


Fig. 2

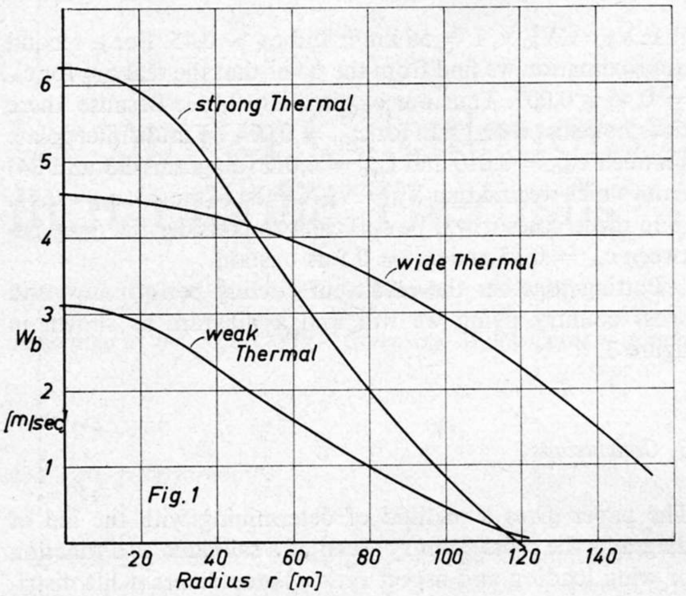


Fig. 1

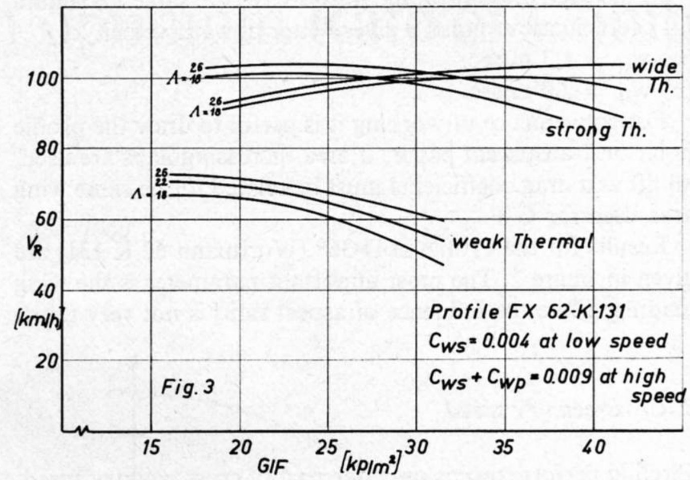


Fig. 3

Zusammenfassung

Die vorliegende Arbeit beschreibt eine Methode zur Bestimmung der Geschwindigkeit im Überlandflug auf Grund der Flächenbelastung, des Seitenverhältnisses und der Profilpolaren für drei Typen von Thermikschläuchen. Dabei ist die Benutzung für Profile mit Klappen möglich.

Die Formel für die Sinkgeschwindigkeit in einer Kurve wird in (1) gegeben, und die erste Zeichnung zeigt die Variation mit dem Radius. Die zweite Zeichnung erläutert, wie der beste Radius und die korrespondierende Geschwindigkeit durch graphische Subtraktion der Sinkgeschwindigkeit von der Steiggeschwindigkeit des Thermikschlauchs errechnet werden kann. Die Berechnungen der besten Steiggeschwindigkeit W_{st} des Segelflugzeuges für verschiedene Λ und G/F und für die drei Typen von Thermikschläuchen wurde in einem Computer vorgenommen und die Ergebnisse als parametrische Diagramme dargestellt, wie sie sich in der dritten Zeichnung sowie in Figur A5 usw. finden. Figur 2 für die D-36 zeigt, dass die Flächenbelastung wichtiger ist als das Seitenverhältnis.

Die Diagramme B 1 usw. der Form wie in der vierten Zeichnung erlauben es, die Geschwindigkeit im Überlandflug zu bestimmen, wenn einmal W_{st} gefunden ist. Wenn C_{wp} nicht konstant ist, sondern mit C_A variiert, wie in der fünften Zeichnung dargelegt, so muss eine zweite Annäherung vorgenommen werden.

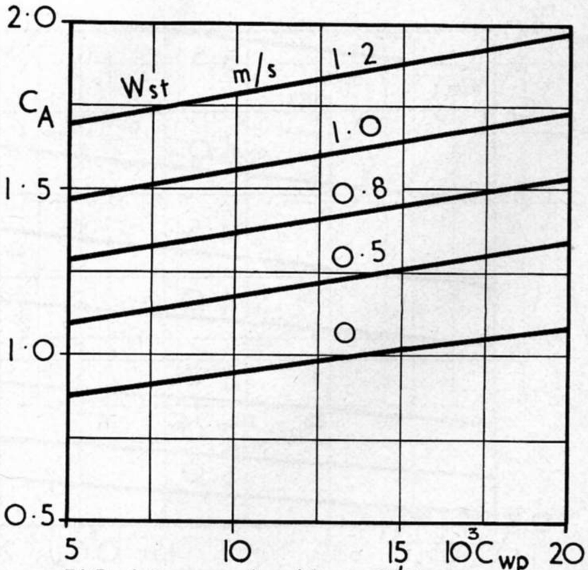


FIG. A5 $\Lambda = 18$ G/F = 30
WEAK THERMAL

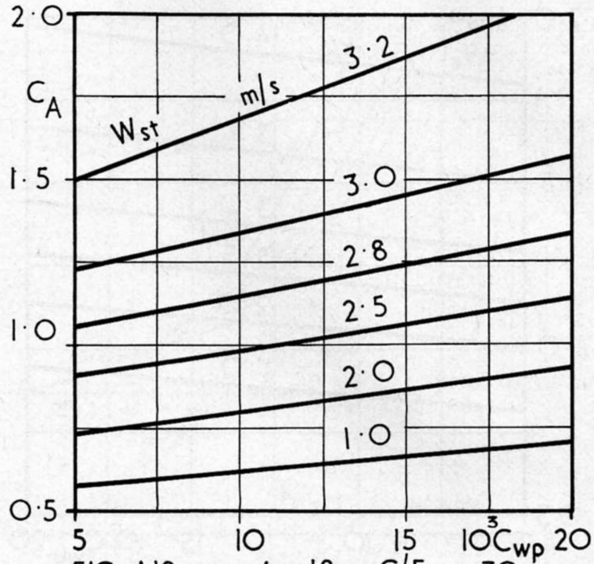


FIG. A12 $\Lambda = 18$ G/F = 30
WIDE THERMAL

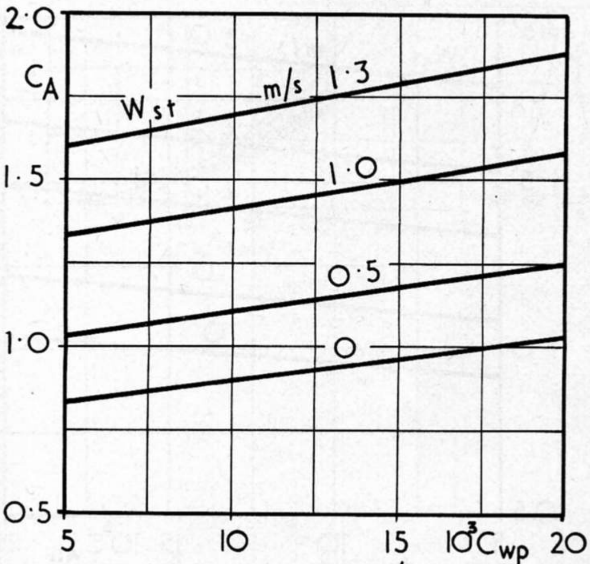


FIG. A6 $\Lambda = 22$ G/F = 30
WEAK THERMAL

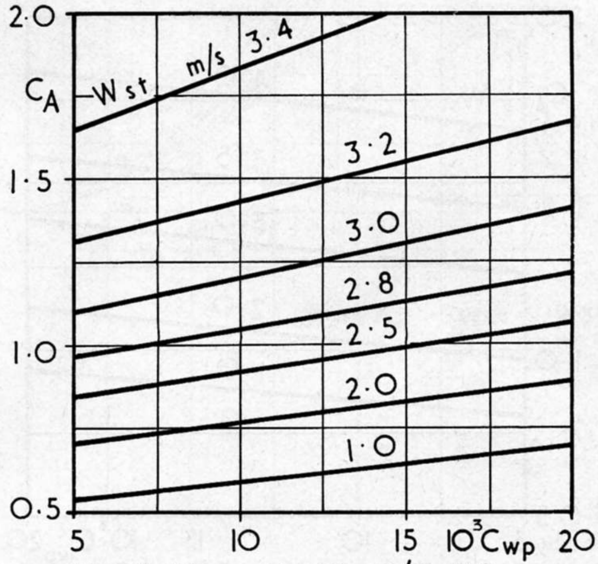


FIG. A 13 $\Lambda = 22$ G/F = 30
WIDE THERMAL

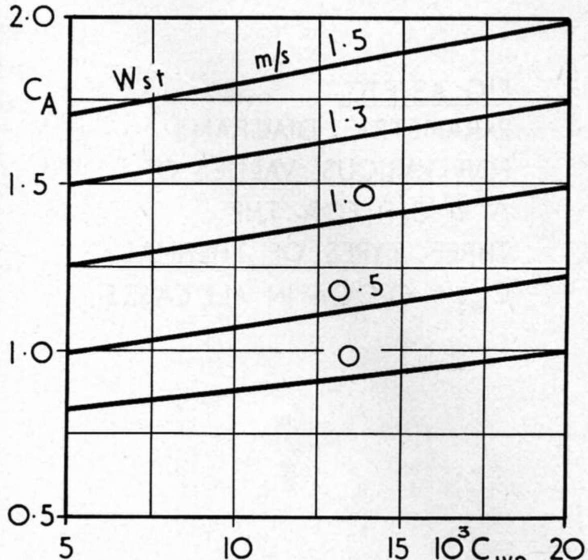


FIG. A7 $\Lambda = 26$ G/F = 30
WEAK THERMAL

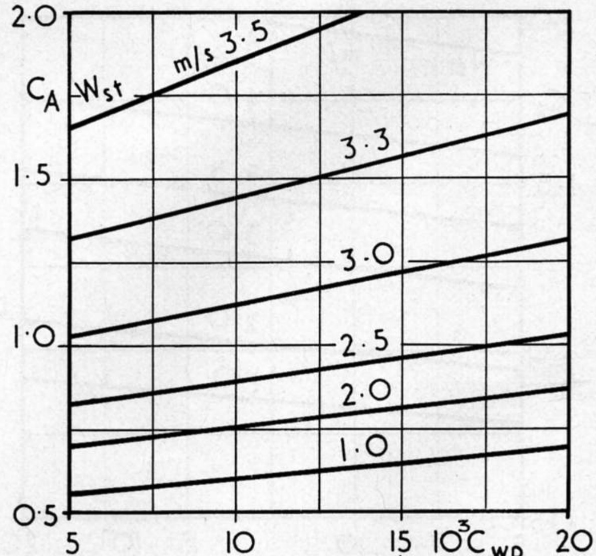


FIG. A 14 $\Lambda = 26$ G/F = 30
WIDE THERMAL

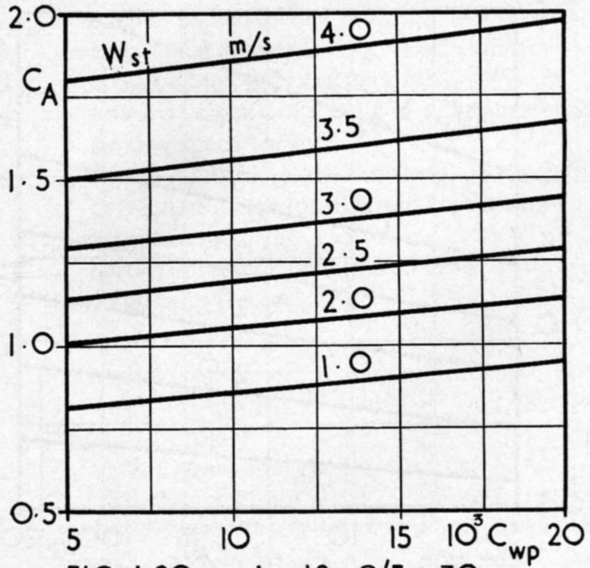


FIG. A 20 $\Lambda = 18$ G/F = 30
STRONG THERMAL

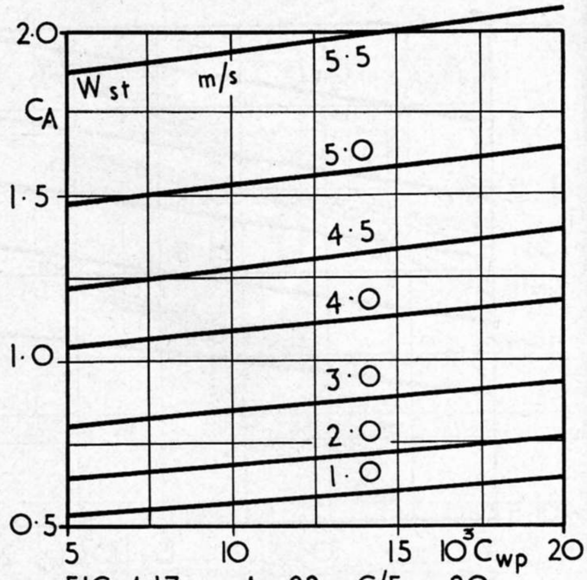


FIG. A 17 $\Lambda = 22$ G/F = 20
STRONG THERMAL

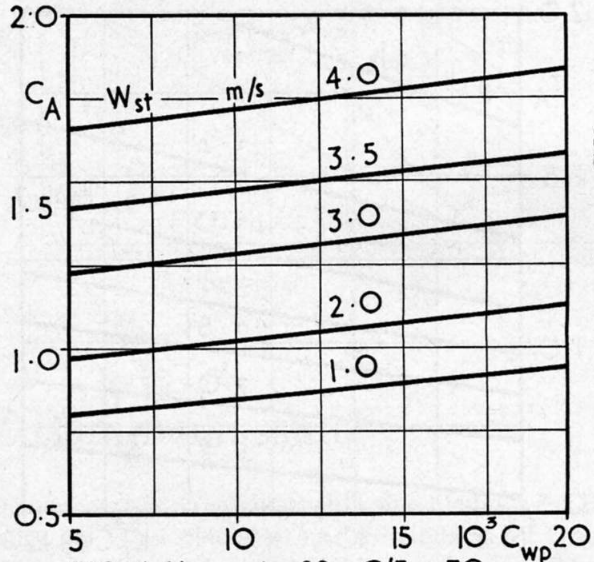


FIG. A 21 $\Lambda = 22$ G/F = 30
STRONG THERMAL

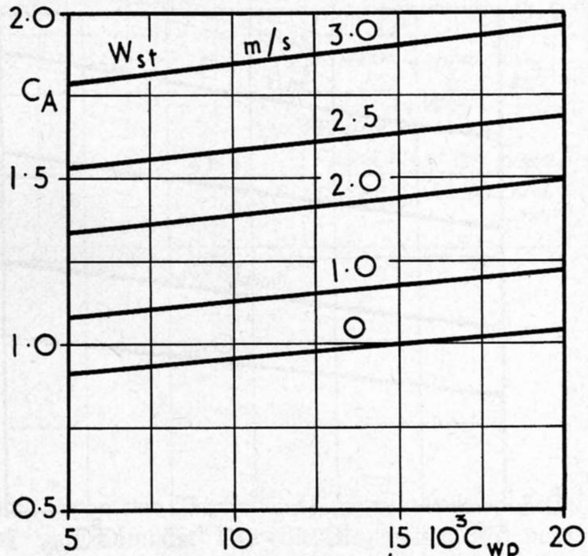


FIG. A 23 $\Lambda = 22$ G/F = 40
STRONG THERMAL

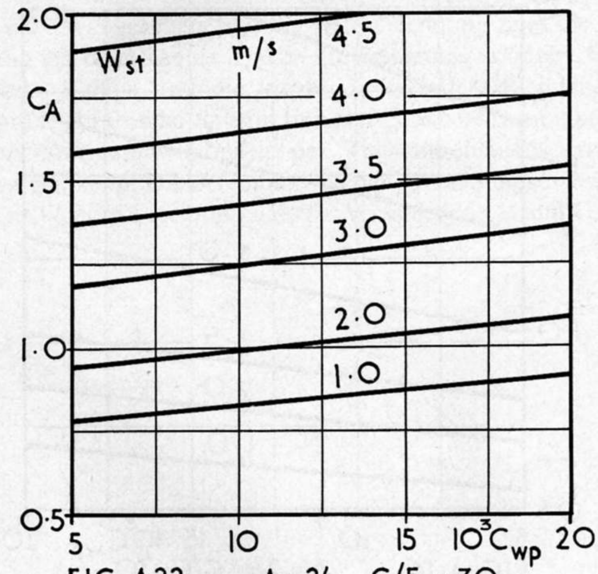
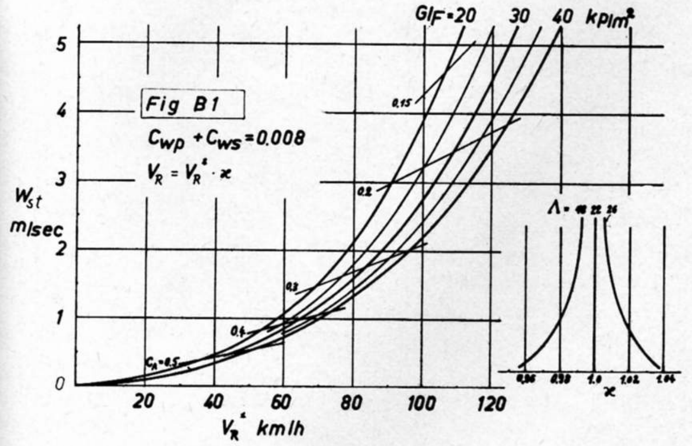
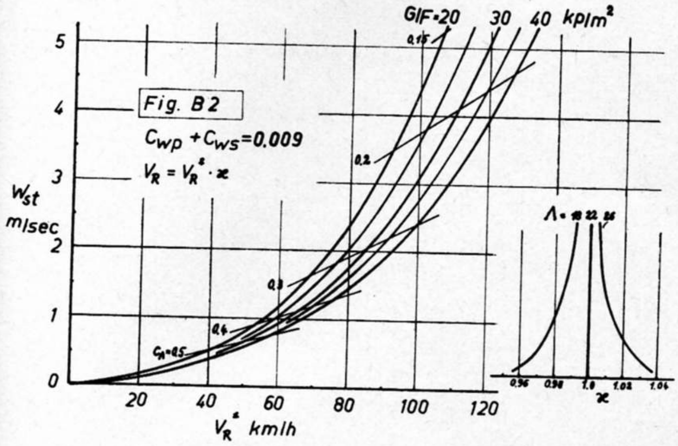
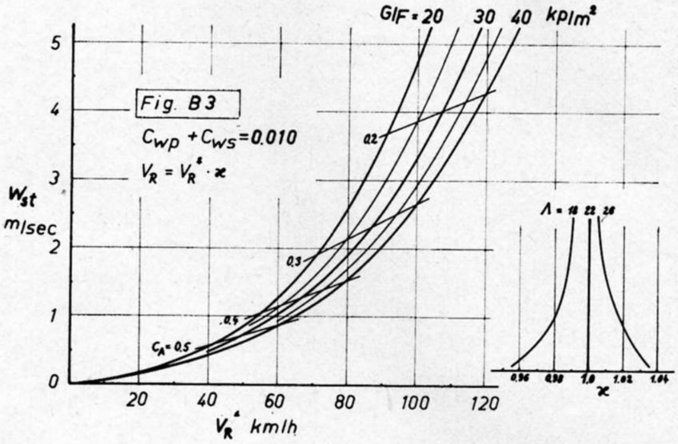


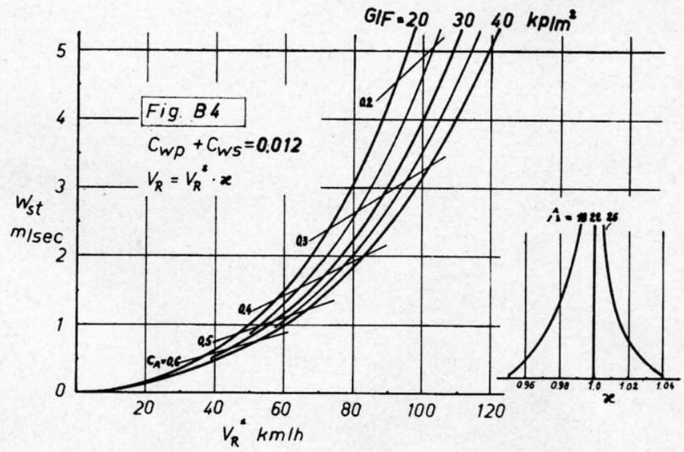
FIG. A 22 $\Lambda = 26$ G/F = 30
STRONG THERMAL

FIG. A5 ETC.
PARAMETRIC DIAGRAMS
FOR VARIOUS VALUES OF
Λ & G/F FOR THE
THREE TYPES OF THERMAL.
$C_{ws} = 0.004$ IN ALL CASES.









(Swiss Aero-Revue 6/1968)