A «Static» Evaluation of the Manoeuvring Tail Load for Instantaneous Unchecked Longitudinal Manoeuvres of Sailplanes

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Introduction

The dynamic response of sailplanes to abrupt longitudinal manoeuvres has been studied by the author in two previous papers [1, 2]. Checked and unchecked manoeuvre cases have been found to be covered by the instantaneous unchecked manoeuvre, related to the same increment of the load factor.

A simple expression for the incremental aerodynamic tail load corresponding to the unchecked instantaneous manoeuvre was derived from the classical «dynamic» equations of the vertical and pitching motion [1].

It is shown here how the same expression can be derived from «static» considerations. This is possible owing to the aperiodic (or quasi-aperiodic) type of response occurring in the case of sailplanes [1]*.

The «static» derivation of the tail load formula

Let us consider the sailplane in the equilibrium condition corresponding, for instance, to point A_1 ($V = V_A$, $n = 1$) (fig. 1) on the n-V manoeuvre envelope. The equilibrium of forces in the vertical direction and of moments about the pitching axis according to the scheme of fig. 2 is written as follows:

$$L_w + L_t = W$$
$$L_w \cdot x_{CG} + M_o - L_t \cdot l_t = 0 \qquad (1)$$

where:

L_w — wing lift
L_t — tail lift
W — sailplane weight
x_{CG} — longitudinal distance of sailplane CG from wing aerodynamic center
l_t — distance of horizontal tail from sailplane CG
M_o — wing aerodynamic moment about the wing a.c.

(the positive sense of forces, moments and lenghts is as indicated in the figure).

The system of equations (1) can be solved with respect to L_t, which is the tail balance load in the flight condition corresponding to point A_1 of the n-V envelope (fig. 1):

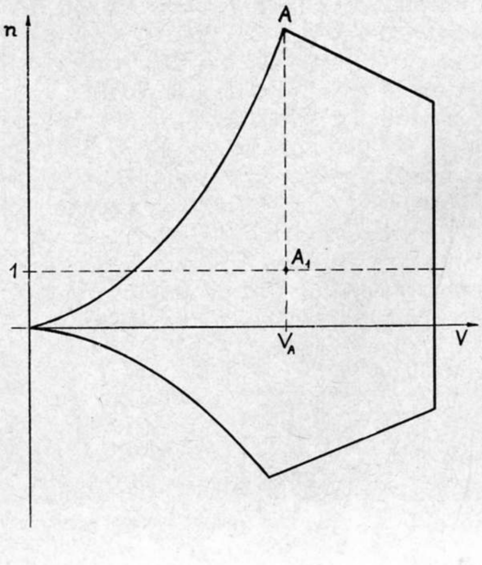
$$L_t = \frac{W \cdot x_{CG} + M_o}{l_t + x_{CG}} \qquad (2)$$

In this condition, an elevator deflection η_{trim} is required, which can be calculated by writing:

$$L_t = k_t q S_t a_t \times \qquad (3)$$
$$\left[\left(1 - \frac{d\epsilon}{d\alpha_w}\right)\alpha_{w_1} - i + \frac{\delta a_t}{\delta \eta}\eta_1\right]$$
$$= c_1 \cdot \alpha_{w_1} - c_2 \cdot i + c_3 \eta_1$$

$$\eta_{trim} = \eta_1 = \frac{1}{c_3}\left(L_t - c_1 \cdot \alpha_{w_1} + c_2 \cdot i\right) \qquad (4)$$
$$= \frac{1}{c_3}\left(\frac{W \cdot x_{CG} + M_o}{l_t + x_{CG}} - c_1 \cdot \alpha_{w_1} + c_2 \cdot i\right)$$

Fig. 1



where:

k_t — tail efficiency ($k_t \cdot q$ = dynamic pressure at the tail)
q — $1/2 \rho V^2$
S_t — horizontal tail surface
a_t — horizontal tail lift curve slope
$1 - \frac{d\epsilon}{d\alpha_w}$ — downwash factor at the tail
α_w — wing angle of attack (with respect to wing zero lift direction)
i — tail setting (angle between the zero lift directions of the wing and of the horizontal tail with neutral elevator)
$\frac{\delta a_t}{\delta \eta}$ — rate of change of tail angle of attack with elevator deflection
η — elevator deflection
c_1 — $k_t q S_t a_t (1 - d\epsilon/d\alpha)$
c_2 — $k_t q S_t a_t$
c_3 — $k_t q S_t a_t \frac{\delta a_t}{\delta \eta}$

Let us consider now that the elevator is given an instantaneous upward deflection $\Delta\eta = \eta_2 - \eta_1$ (which is then mantained). An aerodynamic tail incremental load ΔP will take place instantaneously (disregarding aerodynamic lag)

$$\Delta P = k_t q S_t a_t \frac{\delta a_t}{\delta \eta} \Delta\eta \qquad (5)$$

which produces a vertical acceleration $\Delta P/W$ and a rotational acceleration about the y pitching axis $\Delta P \cdot l_t/J_y$, where J_y is the sailplane moment of inertia about the y-axis.

The dynamic response of the sailplane to this abrupt manoeuvre is such that, after a short time, a new steady state of equilibrium will be attained on a curved path, corresponding, for instance, to point A in the n-V envelope (fig. 1). The equilibrium equations become the following (fig. 3):

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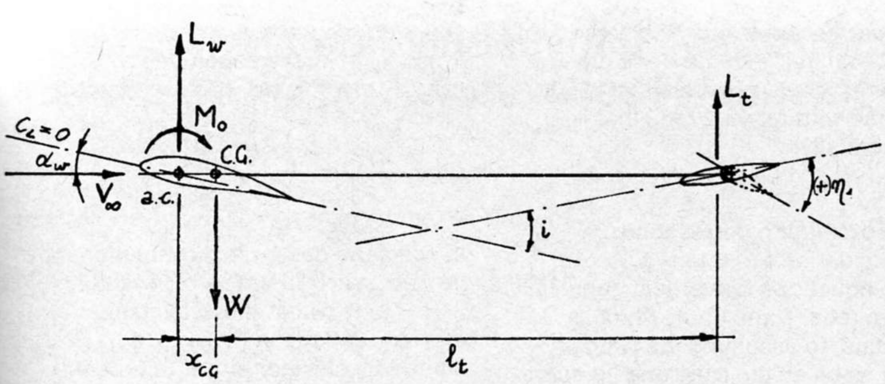

Fig. 2

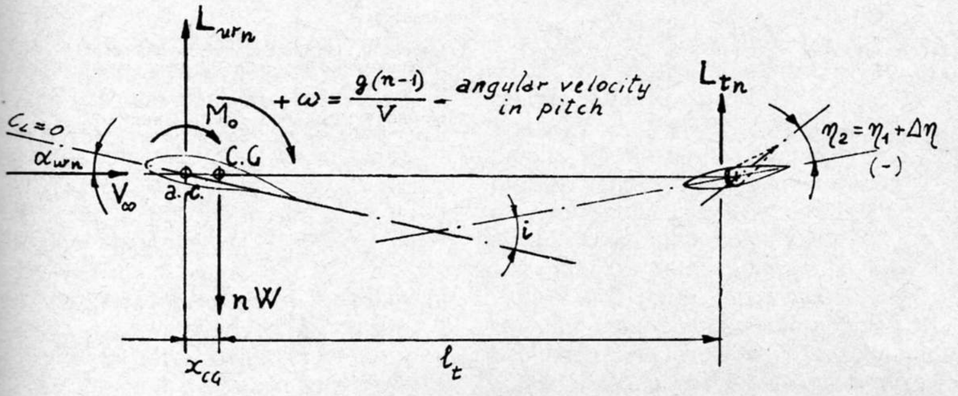

Fig. 3

$$L_{wn} + L_{tn} = nW$$
$$L_{wn} \cdot x_{CG} + M_o - L_{tn} \cdot l_t = 0 \qquad (6)$$

From equations (6) L_{tn} can be derived, which is the tail balance load in the new equilibrium condition of steady accelerated flight

$$L_{tn} = \frac{n \cdot W \cdot x_{CG} + M_o}{l_t + f} \qquad (7)$$

L_{tn} can be expressed as the algebraic sum of three loads: one component load due to the incidence of the tailplane (or stabilizer), one to the elevator deflection and the third one due to the angular velocity in pitch:

$$L_{tn} = k_t q S_t a_t \times \qquad (8)$$
$$\left[\left(1 - \frac{d\epsilon}{da}\right) a_{w_n} - i + \frac{\delta a_t}{\delta \eta} \eta_2 + \frac{l_t \cdot g}{V^2} (n-1) \right]$$
$$= (c_1 \cdot a_{w_n} - c_2 \cdot i) + c_3 \cdot \eta_2 + c_4 (n-1)$$

where $\eta_2 = \eta_1 + \Delta\eta$ is the new elevator angle (η_{trim} for the new equilibrium condition), $c_4 = l_t \cdot g/V^2$.
It is generally admitted that, during such abrupt manoeuvres as considered here, the airspeed remaines unchanged. V and q in eq. 8, therefore, mantain the same values as in the initial condition (eq. 3).
Obviously, the difference between L_{tn} (eq. 8) and L_t (eq. 3) gives the aerodynamic tail incremental load that takes place when the aircraft is brought from the initial equilibrium condition (n = 1) to the final post-manoeuvre new equilibrium condition. Of course, this evaluation can be made more generally from any initial equilibrium condition ($n = n_1$) to any final post-manoeuvre equilibrium condition ($n = n_2$). This simply requires to apply twice eq. 8, therein introducing $n = n_1$ and $n = n_2$, respectively.
The difference $\Delta L_t = L_{tn} - L_t$ or, more generally,

$$\Delta L_t = L_{tn_1} - L_{tn_2} \quad ,$$

results:

$$\Delta L_t = c_1 \cdot \Delta a_w + c_3 \cdot \Delta\eta + c_4 \cdot \Delta n$$
$$= c_1 \cdot \Delta a_w + \Delta P + c_4 \cdot \Delta n \qquad (9)$$

$\Delta P = C_3 \Delta\eta$ n is the aerodynamic incremental load due to elevator deflection.
This load ΔP has not much significance, in general, on motorplanes.
In fact, the aerodynamic response of an aircraft to an abrupt longitudinal manoeuvre is a damped oscillation of a_w, L_t and n with time. A peak value of n (n_{max}) takes place after a short time, the new steady value n_∞ being reached after a few damped oscillations. In the particular case of sailplanes, however, due to their geometric and mass characteristics, it was shown [1] that the dynamic response is aperiodic (or quasi-aperiodic). This means that n_{max} practically coincides with the assymptotic value n_∞, pertinent to the new steady state accelerated flight condition at the new elevator angle: $n_{max} \cong n_\infty$.
In such a case, therefore, ΔP contained in eq. 9 is exactly the aerodynamic incremental load due to the incremental elevator deflection $\Delta\eta$ that we want to determine.
It is evident, in any case, that a particular Δn corresponds to each value of $\Delta\eta$ and, therefore, of ΔP. The correlation between ΔP and Δn is needed in order to evaluate the aerodynamic tail incremental load capable to produce the prefixed Δn.
By subtracting eq. (2) from eq. (7), we obtain:

$$\Delta L_t = L_{tn} - L_t$$
$$= \frac{nWx_{CG} + M_o}{l_t + x_{CG}} - \frac{Wx_{CG} + M_o}{l_t + x_{CG}}$$
$$= \frac{x_{CG}}{l_t + x_{CG}} \Delta n \cdot W \qquad (10)$$

Δa_w, the incremental wing incidence, appearing in eq. (9), can be expressed as follows:

$$\Delta L_w = q\,S\,a \cdot \Delta a_w = c_5 \cdot \Delta a_w \qquad (c_5 = q\,S\,a)$$

From vertical equilibrium eqs. (1) and (6), and from eq. (10):

$$\Delta L_w = \Delta n \cdot W - \Delta L_t \qquad (11)$$
$$= \Delta n \cdot W - \frac{x_{CG}}{l_t + x_{CG}} \Delta n \cdot W$$
$$= \Delta n \cdot W \frac{l_t}{l_t + x_{CG}} = c_5 \Delta a_w$$
$$\Delta a_w = \frac{1}{c_5} \frac{l_t}{l_t + x_{CG}} \Delta n \cdot W$$

From eqs. (9), (10) and (11), therefore:

$$\Delta P = \Delta L_t - c_1 \Delta \alpha_w - c_4 \Delta n \qquad (12)$$

$$= \frac{x_{CG}}{l_t + x_{CG}} \Delta n \cdot W - \frac{c_1}{c_5} \frac{l_t}{l_t + x_{CG}} \Delta n \cdot W$$

$$- c_4 \Delta n$$

$$= \Delta n \cdot W \frac{l_t}{l_t + x_{CG}} \left[\frac{x_{CG}}{l_t} - \frac{c_1}{c_5} - \frac{c_4}{W} \right]$$

or

$$\Delta P = \Delta n \cdot W \frac{l_t}{l_t + x_{CG}} \times \qquad (13)$$

$$\left[\frac{x_{CG}}{l_t} - k_t \frac{S_t}{S} \frac{a_t}{a} \left(1 - \frac{d\epsilon}{d\alpha}\right) - S_t a_t (l_t + x_{CG}) \frac{k_t \varrho g}{2W} \right]$$

This expression is practically the same as eq. (33) in [1]. In the derivation of the latter it was assumed that x_{CG} is so small with respect to l_t, that it is practically

$l_t \cong l_t + x_{CG}$, $l_t / (l_t + x_{CG}) \cong 1$.

This assumption corresponds to considering $L_w + L_t \cong L_w$ in the equations for vertical equilibrium (eqs. 1 and 6) or, which is the same, to assuming the wing lift curve slope as the sailplane lift curve slope (sailplane with a fixed elevator deflection, of course).
The above eq. (13), therefore, is more general. In practice, however, it yields the same $\triangle P$ values as the expression of [1], i.e.:

$$\Delta P = \Delta n \cdot W \left[\frac{x_{CG}}{l_t} - k_t \frac{S_t}{S} \frac{a_t}{a} \left(1 - \frac{d\epsilon}{d\alpha}\right) - k_t S_t a_t l_t \frac{\varrho g}{2W} \right]$$

The incremental elevator deflection $\triangle\eta = \eta_2 - \eta_1$ corresponding to the $\triangle P$ given y eq. (13), is given by

$$\Delta\eta = \frac{\Delta P}{c_3} = \frac{\triangle P}{k_t q S_t a_t \delta a_t / \delta\eta}$$

In sailplane design this evaluation may be necessary, in order to verify that $\eta_1 + \triangle\eta$ is within the available elevator deflection ($\pm\eta_{\max}$), as limited by elevator stops. In fact, it is generally an airworthiness requirement that the elevator is sufficiently powerful to allow any manoeuvre within the boundary of the n-V envelope.

References

1. Morelli, P.: On the Dynamic Response of Sailplanes to Longitudinal Manoeuvres (Aero-Revue, 5–61/1967).

2. Morelli, P.: Tail Loads due to Abrupt Longitudinal Manoeuvres (Aero-Revue, 10/1968).