Computer Analysis of the Performance of 15 m Sailplanes Using Thermals with Parabolic Velocity Distributions

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Summary
A computer programme has been devised by H. C. N. Goodhart which will calculate the performance of a sailplane of given characteristics in straight and circling flight. It will also calculate the optimum rate of climb in thermals having parabolic velocity distributions and hence will derive the corresponding average cross-country speed. Allowance is made for Reynolds number effects.
A family of 15 m sailplanes was devised with aspect ratios of 12, 16, 20, 24 and 28, each at various weights, making a total of 36 sailplanes. Thermals having strengths between 4 and 12 knots and radii between 300 and 1,000 feet were considered (25 thermals in all).
It was concluded that, under most conditions, the average cross-country speed is not very sensitive to aspect ratio provided that the weight is optimised. For weak thermals, the best aspect ratio is high, but the corresponding weight is too low to be practicable. The best compromise sailplane has quite a low aspect ratio (about 16) but needs to be able to carry a large amount of ballast.

Units
The original calculations were carried out in mainly English units, often using round numbers. In the earlier part of the paper, equivalent metric units are also given as an order-of-magnitude guide but in drawing conclusions, it seemed simplest to relate the results to the original, English figures.

The Computer Programme
The input data are as follows:
(a) Geometry of the sailplane:
- Wing span
- Wing area (or aspect ratio)
- Horizontal tail area.
- Vertical tail area
(b) Weight (or wing loeading).
(c) Aerodynamic characteristics:
- Wing section characteristics as given by the C_D-C_L curves for various Reynolds numbers
- Vertical and horizontal tail drag coefficients corresponding to a Reynolds number of 10^6 (based on mean tail chords)
- «Miscellaneous drag». This is the drag of all parts other than the surfaces mentioned above. It is expressed in lb force at a speed of 100 ft/sec
- Induced drag factor (assumed constant)
- Climbing lift coefficient (assumed constant).

Provision is also made for sailplanes of the «Sigma» type, by inserting a «flap extension factor» and separate induced drag factors appropriate to the climbing and crusing configurations.
(d) Thermal characteristics:
The radius and central strength as defined in figure 1, assuming a parabolic velocity/radius relationship. In the most adverse cases considered, the radius of turn of the sailplane is about $^2/_3 \times$ thermal radius.
The computer then provides:
(1) The straight flight performance curve of the sailplane. This takes into account the effect of Reynolds number on the wing C_D-C_L characteristics, interpolating or extrapolating as necessary. At a given C_L, the wing C_D was assumed to be inversely proportional to $\sqrt{R_e}$.[1]
This assumption does not permit much extrapolation beyond the experimental points to which the curve is fitted, and the printout states when extrapolation has occurred, as a guide to its reliability. The tail C_D-R_e relationship was assumed to be hyperbolic.
The print-out tabulates the following quantities as functions of forward speed (in knots):
Rate of sink (knots)
Glide ratio
Wing C_D
Induced drag (lb)
Wing profile drag (lb)
Tail profile drag (lb)
Miscellaneous drag (lb)
Total drag (lb)
Reynolds number.

[1] This relationship is strictly only applicable to laminar boundary layers and is only used for extrapolation. See [1], pp. 2–4, para 3.

(2) Turning flight performance based on the straight flight performance at the chosen lift coefficient.
The print-out states the forward speed (knots) and the rate of sink (knots) appropriate to the chosen C_L and then tabulates, as a function of radius of turn (ft), the following quantities:
Rate of sink (knots)
Angle of bank (degrees)
Forward speed (knots).
No allowance is made for changing Reynolds number, it being simply assumed that the rate of sink and forward speed are proportional to $(\sec \phi)^{3/2}$ and $(\sec \phi)^{1/2}$ respectively.
(3) Cross-country performance assuming that the sailplane circles at that radius which gives maximum rate of climb in a given thermal [2] and then flies at the appropriate optimum speed between the thermals. For each combination of thermal strength and radius, the print-out lists:
Average cross-country speed (knots)
Cruise speed (knots)
Achieved rate of climb (knots)
Bank angle in circling flight (degrees)
Forward speed in circling flight (knots).

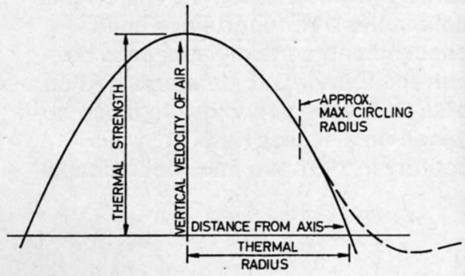

Fig. 1

Notes on this Programme
In calculating the straight-flight performance, the following are not taken into account:
- The effect of Reynolds number on the profile drag of parts of the sailplane other than the wing and tail
- The effects of trim drag
- Effects which may occur in practice but which are virtually impossible to analyse, such as fortuitous boundary layer separations at high lift coefficients which may increase the profile drag and modify the induced drag factor.

One would imagine that these omissions would tend to give rather optimistic performance figures for a given sailplane at a given weight. However, the performance also depends on a suitable choice of the figure for the «miscellaneous drag». If a realistic figure is taken, it can be regarded as including most of the above effects. Although in theory they may not vary with speed in a simple speed-squared

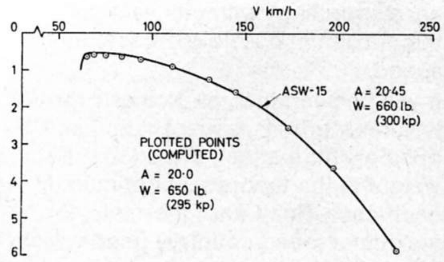

Fig. 2

fashion, in practice the results seem to be satisfactory. Analysis of actual sailplane performance enables a figure for miscellaneous drag to be deduced: for very clean designs of about 18 m span, it is of the order of 3.0 lb at 100 ft/sec. For the present purposes, 3.5 lb was taken so as not to be unduly optimistic.

Figure 2 compares the claimed maker's performance of the ASW-15 with that calculated by the present method for a sailplane of nearly the same aspect ratio and weight.

The Computations

For the present purposes, a «family» of 15 m sailplanes was defined. All of the sailplanes were assumed to have substantially the same fuselage, giving the same miscellaneous drag (3.5 lb at 100 ft/sec) and the same tail moment arm (15 ft). Aspect ratios of 12, 16, 20, 24 and 28 were assumed.

It was then necessary to make some plausible assumptions about tail size. Possibly, the simplest procedure would be to assume a constant horizontal tail volume ratio, which would give an approximately constant range of permitted centre-of-gravity positions measured as a fraction of the chord. However, it will eventually be shown that the empty weights of the proposed family of sailplanes do not very much so, for a given range of pilot weight, the corresponding range of laden CG positions will be roughly constant in absolute terms (i.e. in inches or mm aft of a datum). As a fraction of the mean chord, the range of CG positions would increase as the aspect ratio increases. The required tail volume ratio would then correspondingly increase with aspect ratio. For a constant tail volume ratio, the tail area would be proportional to the square of the wing area (for constant wing span, constant tail moment arm and variable aspect ratio). For the present purposes, to achieve a tail volume ratio which increases with aspect ratio without indulging in lengthy calculations, it has simply been assumed that the tail area is proportional to the wing area. The datum for the tail area calculations is a typical case: a tail volume ratio of 0.5 for the sailplane with an aspect ratio of 16, the tail moment arm being 15 ft (4.58 m) in all cases. Likewise, a constant vertical tail volume coefficient is not required, since some of the vertical tail area – a constant amount – is required to balance out the destabilising effect of the fuselage. In the interests of simplicity, the vertical tail area has been taken as a constant proportion (0.75) of the horizontal tail area.

Subject to the above assumptions, the geometry of each sailplane of the family has now been defined.

For each sailplane of the family, a series of weights was chosen. They were, to some extent, chosen as the computations were carried out in order to provide significant maxima. In some cases, the weights used for the calculations are improbable, since it is obviously useful to carry such computations beyond the boundaries of current practice. Even so, the optimum sailplane has not always been achieved under the more extreme combinations of thermal strength and radius.

Table I summarizes the geometry of the family of sailplanes and lists the weight taken for each sailplane. In all, 36 sailplanes were considered.

Table I

Aspect ratio	Wing area ft²	Horiz. tail area ft²	Vertical tail area ft²	Weights lb
12	202	20.8	15.6	550 to 1,050 6 values
16	152	15.6	11.7	450 to 950 7 values
20	121	12.4	9.3	400 to 850 7 values
24	101	10.4	7.8	350 to 750 7 values
28	86.5	8.9	6.7	300 to 850 9 values

Other assumed characteristics were as follows:

- Wing section: Wortman FX 61–163. The experimental properties of the section were used, the Reynolds number being based on the wing geometric mean chord [3]
- Horizontal tail profile drag coefficient: 0.0060
  0.0065
- Vertical tail profile drag coefficient:
- Induced drag coefficient (climb and cruise): 1.05
- Climbing lift coefficient: 1.10
  Strictly, the lift coefficient in turning flight corresponding to minimum rate of sink is a function of radius of turn. However, at lift coefficients above 1.10, the profile drag rise is so rapid that the variation of optimum lift coefficient would be small.

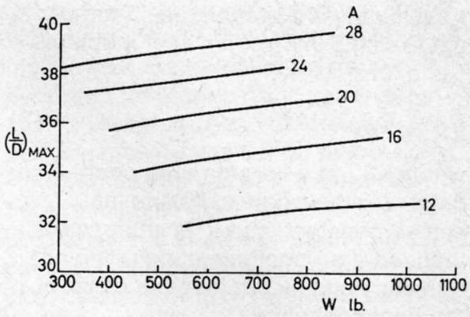

Fig. 3

  One also needs a reasonable speed margin above the stall
- Miscellaneous drag: 3.5 lb (1.59 kp) at 100 ft/sec (30.4 m/sec).

The performance of each sailplane was computed in thermals strengths of 4, 6, 8, 10 and 12 knots (2.06, 3.09, 4.12, 5.15 and 6.18 m/sec).[2] For each thermal strength, thermal radii of 300, 400, 500, 700 and 1000 ft (91.4, 121.9, 152.4), 213.5 and 304.6 m) were considered. The total number of «terminals» was therefore 25.

To summarize, 36 sailplanes were considered in 25 combinations of thermal strength and radius, giving a total of 900 cases.

In this present paper, only the results relating to thermals having radii of 300, 500 and 700 ft are presented, in the interest of brevity.

Effects of Reynolds Number

The effect of Reynolds number is that the profile drag coefficient of the wing decreases as R_e increases at constant lift coefficient.

A consequence is that, for a sailplane of given geometry, the maximum lift/drag ratio improves as the weight increases, since it occurs at progressively higher Reynolds numbers. In figure 3, calculated values of $[L/D]_{max}$ are plotted as functions of weight for various aspect ratios. It will be seen that the variation of $(L/D)_{max}$ is not insignificant.

A further consequence is that the apparent induced drag factor becomes rather large. For these computations, the total drag coefficient is implicitly of the form

$$C_D = C_{Dow}(R_e) + S_T\, C_{DoT}(R_e)/S + C_{D_{misc}} + k\, C_L^2/\pi A \qquad (1)$$

where k is a «true» induced drag factor depending only on the nature of the spanwise lift distribution.

[2] Note that these figures are the central velocities of the air in thermals. The rates of climb achieved are much lower, due to the lower absolute strength at circling radius and the subtraction of the rate of sink of the glider in circling flight.

As a matter of observations, it turns out that this expression is of much the same form as

$$C_D = C_{Do} + k' C_L^2/\pi A, \qquad (2)$$

where C_{Do} is a «profile drag coefficient» for the whole sailplane, assumed constant, and k′ is an apparent induced drag coefficient including not only the actual k but also the effect of the dependence of C_{Dow} and C_{DoT} on R_e.

Figure 4 shows a plot of C_D vs. C_L^1, deduced from the computed performance of the (A=20, W=650 lb) sailplane. Assuming the relationship to be of the form of eq. (2), the value of k′ deduced from the mean slope of the line is 1.26, a considerable increase over the assumed value of k (=1.05).

Practical All-Up-Weights

As noted above, a wide range of weights was chosen for each configuration. In many cases, the weights are impracticably low and, as will be seen, conclusions about optima are greatly influenced by considerations of practical weights.

Practical weights were estimated from the expression given in [4]. For a span of 15 m and assuming the weight of pilot plus parachute and instruments to be 100 kp, the expression for the total weight becomes:

$$W = 152.4 + 4.29\,A + 793/A \text{ kp}.$$

W is plotted against aspect ratio in figure 5. It will be seen that the weight varies from 270 kp (595 lb) at A=12 to 301 kp (663 lb) at A=28.

Consideration of the Computed Performances

Typical results, showing the performance of a sailplane of aspect ratio 20 in 500 ft radius thermals, are presented in figure 6. As expected, there is an optimum weight, giving maximum corss-country speed, for each thermal strength. The optimum weight varies from 460 lb in 4 knot thermals to 810 lb in 10 knot thermals and is slightly beyond the end of the curve for 12 knot thermals. On this plot, the locus of the maxima is nearly a straight line. Since the practical weight for this aspect ratio is 612 lb, it will be seen that the optimum sailplane appropriate to thermal strengths below 6 knots cannot be built at this aspect ratio.

Figure 7 elaborates the previous diagram and shows the computed results for all the aspect ratios and weights considered, again in 500 ft radius thermals. By considering the maxima of all the curves relating to a given thermal strength, it is possible to deduce the optimum aspect ratio which, at the

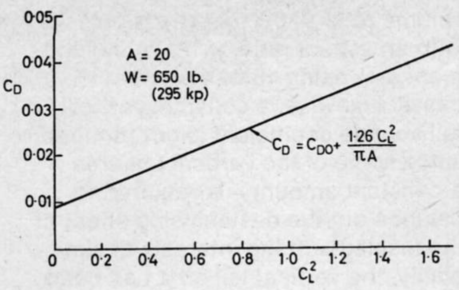

Fig. 4

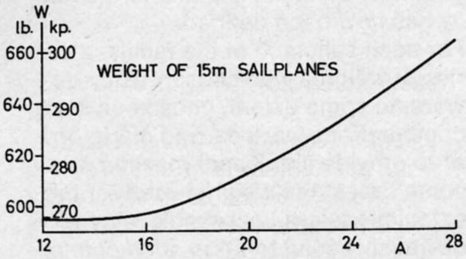

Fig. 5

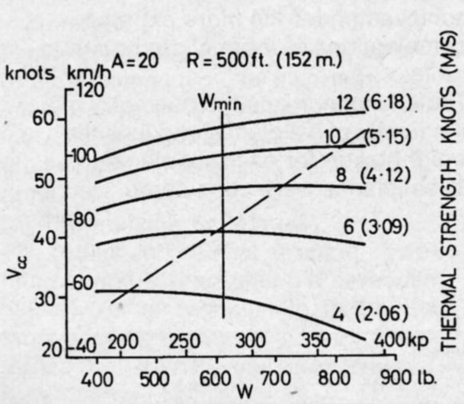

Fig. 6

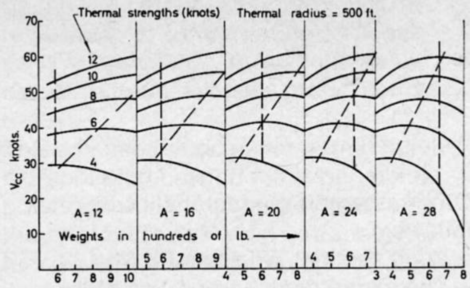

Fig. 7

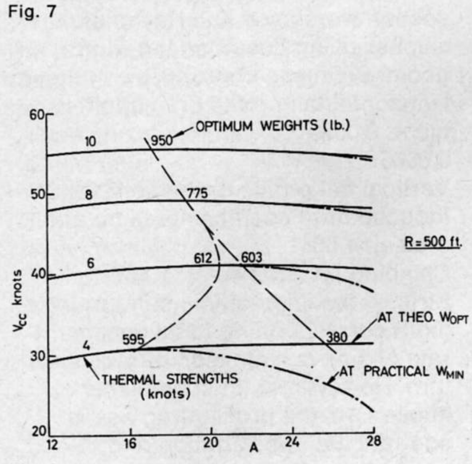

Fig. 8

corresponding optimum weight, gives the maximum possible cross-country speed.

It will be seen that, particularly for the weaker thermals, aspect ratio has surprisingly little effect, provided that the weight is the theoretical optimum in each case. For 4 knot thermals, the maximum cross-country speed varies from 29.3 knots at A=12 to a maximum of 31.1 knots at A=25, becoming 31.0 knots at A=28. It is clear that, assuming current structural techniques, the theoretical optimum sailplane for 500 ft diameter, 4 knot thermals (viz, A=25, W=380 lb) cannot be built. The real «best» sailplane would have an aspect ratio of about 16 and a weight just under 600 lb. The difference in performance between this «best» sailplane and the theoretical but unattainable optimum would be small: about 30.4 knots compared with 31.1 knots, or a loss of 2.2%.

This situation only applies when considering weak thermals. For 6 knot, 500 ft radius thermals, the optimum sailplane is very nearly the practicable one (A=20, W=612 lb). In stronger thermals, weights of the optimum sailplanes are higher than the lowest practical weight, and no problem arises. For example, in 8 knot, 500 ft radius thermals, the optimum sailplane has an aspect ratio of 19 but the optimum weight is about 775 lb, as compared with the practical minimum of 608 lb. The desired weight can be obtained by adding ballast. For thermals of 500 ft radius, the results are summarized in figure 8. The theoretical optimum aspect ratio varies from about 25 for 4 knot thermals to about 17 for 10 knot thermals in a reasonably regular fashion. However, taking practicable weights into consideration, it increases from 16 in 4 knot thermals to 20 in 6 knot thermals and then declines again to 17 in 10 knot thermals. The corresponding weights are also indicated in the diagram.

In 300 ft thermals [fig. 9), the problem is more difficult. In 4 knot thermals, for a given aspect ratio, the theoretical optimum sailplane is lighter than any of those computed. It is clear that the practical optimum sailplane has an aspect ratio under 12. For 6 knot thermals, the theoretically optimum performance is very insensitive to aspect ratio and has a maximum at about A=18. This sailplane is somewhat lighter than is practicable: the real optimum occurs at about A=14. In 8 knot thermals, the theoretical optimum sailplane is also just practicable at A=16. In stronger thermals, the optimum aspect ratio is lower, declining to about 12 in 12 knot thermals and the lightest practicable sailplanes would require ballast to bring them up to the optimum weights (see fig. 10). In 700 ft

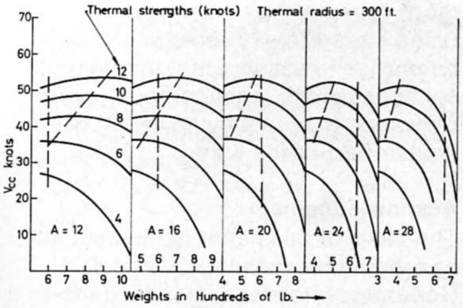

Fig. 9

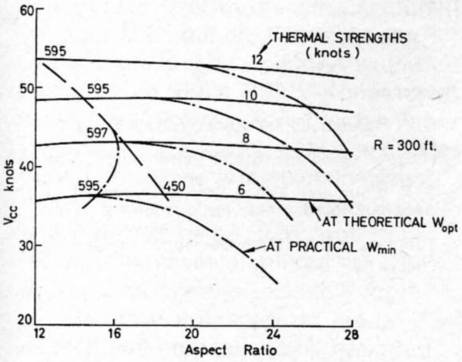

Fig. 10

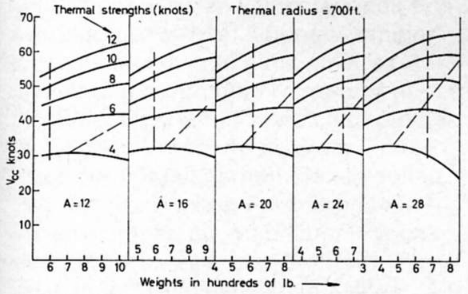

Fig. 11

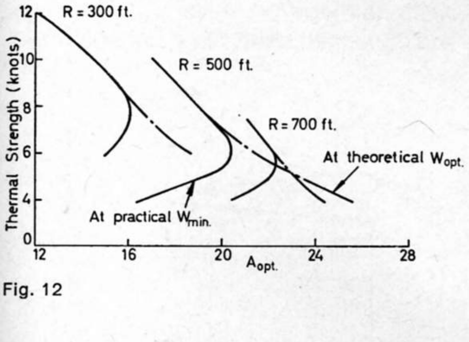

Fig. 12

thermals, there is little problem (fig. 11): only in 4 knot thermals is the practical optimum (A=20) different from the theoretical (A=24). In many cases, the optimum weight is higher than the greatest weight computed.

The effect of thermal size and strength on optimum sailplanes may be summarized as follows:

- For a given radius of thermal, the theoretical optimum aspect ratio increases as the thermal strength decreases
- The theoretical corresponding optimum weight decreases as the thermal strength decreases
- Below a certain thermal strength, the theoretical optimum weight becomes less than that of practicable sailplanes (on the basis of current structural techniques)
- Below this thermal strength, the practical optimum aspect ratio decreases as the thermal strength decreases
- Above this thermal strength the optimum weight is higher than the lowest practicable weight and it may be necessary to add ballast
- For a given thermal strength, the optimum aspect ratio increases as the radius of the thermal increases.

These results are summarized in figure 12.

Conclusions

From these results, it is possible to select the optimum sailplane which provides the highest possible average cross-country speed in thermals of a given strength and radius.

The designer normally wishes to design a sailplane suitable for use in a wide variety of conditions, so as to provide adequate performance in competitions when the weather is poor, to provide the maximum enjoyment to owners in general, and hence to maximise its sales prospects. For example, a sailplane with an aspect ratio of about 22 and a weight of 624 lb would correspond to the optimum for use in thermals of about 5 knots having a radius of 700 ft. In 4 knot, 500 ft radius thermals, its cross-country speed would be about 28.8 knots, or about 5% less than the maximum attainable under these conditions. In brief, high aspect ratios are only suited to thermals of moderate strength and large radius. In both weak and strong thermals, particularly of small or moderate radius, a more modest aspect ratio is appropriate.

Remembering that a sailplane can be ballasted to a weight higher than the practical minimum, it would seem prudent to design for poor thermal conditions. By adding ballast, the same configuration of machine would then be near the optimum for strong conditions. In thermals of medium strength (or large radius) it would depart appreciably from the optimum configuration. However, it is most important to appreciate that the penalty for having an aspect ratio lower than the optimum, but with a weight which is both practicable and the optimum for that aspect ratio, is significantly less than having an aspect ratio higher than the optimum for those conditions, coupled with a practicable weight which is higher than the optimum for that aspect ratio. In other words, particularly in small thermals, the «practicable» performance curves fall far below the «theoretical» curves at the higher aspect ratios (see fig. 8 and 10).

Ultimately, the choice of a suitable aspect ratio will depend on the designer's assessment of the weather conditions to which the sailplane is to be matched. Consider, however, a sailplane with an aspect ratio of 16 and a weight of 597 lb (the minimum practicable weight for this aspect ratio). Assume also that it is designed so that large amounts of ballast can be accommodated. In the weakest and smallest hermals considered [4 knots, 300 ft radius) its performance would be significantly worse than the maximum attainable but the latter would involve such a very low aspect ratio that its performance under better conditions would be severely prejudiced. Nevertheless, its cross-country speed is 25 knots even under these poor conditions. In thermals of 6 knots, 300 ft radius and 4 knots 500 ft radius, it is very close to the optimum. In thermals of 4 knots, 700 ft radius, its average speed is 32.2 knots compared with a practical maximum of 32.7 knots, a loss of only 1.5%.

In 300 ft radius thermals, the performance is again slightly worse than optimum (by 1.4%) in 6 knot thermals. In stronger thermals at this radius, suitably ballasted, it is either the optimum sailplane or, in terms of average speed, almost indistinguishable from optimum. In 12 knot thermals, it would need to be ballasted to an all-up weight of 710 lb.

In 500 ft radius thermals, it is the optimum sailplane when the strength is 4 knots and, ballasted to about 900 lb, in 10 knot thermals. In 12 knot thermals it would seem to be very close to the optimum at a slightly higher weight.

In 700 ft radius thermals, its aspect ratio is always lower than optimum but, given suitable ballast, the performance penalty is small. When the strength is 4 knots, ballasted to 650 lb it achieved 32.3 knots as against 32.6 knots (corresponding to A=18), a loss of 1%. In very strong thermals (12 knots) its optimum weight is more than the maximum computed (950 lb), but at this weight its average speed is 64 knots compared with a theoretical maximum of probably 65.5 knots (2.3% deficit).

This machine, with an aspect ratio of 16. is therefore very close to the practical optimum over a very wide range of conditions. Only in weak, small thermals is it noticeably worse than optimum and even then its performance, in absolute terms, is quite good. Under most conditions, its perform-

ance is within about 1.5% of the maximum practicable figure.
In broad terms, it therefore seems that lower aspect ratios than those currently in fashion are, in fact, of greater utility, provided that provision is made for the carriage of a large amount of ballast. If the sailplane is to operate at near-optimum conditions in large, strong, thermals, it should be possible to increase its AUW by something over 50%. Such a large amount of ballast would, of course, have to be carried in the wings and the provision of suitable tanks, valves, etc. would itself tend to increase the unballasted weight above the figures assumed in this paper. This would tend to accentuate the need for a lower aspect ratio.
The maximum L/D for a sailplane with an aspect ratio of 16 is about two units less than for an aspect ratio of 20. However, the overall performance of the former is better than that of the latter under many conditions, due to its superior climbing ability. It is clear that the wording of a brochure for the lower aspect ratio sailplane would need careful consideration.
All of the foregoing relates to sailplanes using FX-61 series wing sections. The conclusions will be different if sections are used which have significantly different operating lift coefficients from those of the FX-61 sections. In particular, if the lift coefficient in circling flight can be increased, it seems likely that in increase in aspect ratio can be accepted.
Finally, it is important to appreciate that these results only relate to sailplanes flying in thermals of the assumed «parabolic» type. They are very sensitive to thermal radius and hence one would expect them also to be sensitive to the shape of the velocity distribution. There are some grounds for believing that the velocity distribution near the centre of a real thermal is likely to be flatter than parabolic. Until more work has been done on the detailed structure of thermals, it would be unwise to assume that the numerical conclusions reached here have any relevance to actual flying in any particullar part of the world.

Acknowledgement
The basis of this paper is the computer programme developed by H. C. N. Goodhart, who also supervised the computations relating to the family of 15 m sailplanes devised by the author and provided much valueable advice and comment. I am most grateful to him.

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