Influence of Wing Bending Flexibility on Sailplane Loads due to Discrete Gusts

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1. Introduction

The modern methods of computing sailplane loads under turbulent conditions require some simplifying assumptions e.g. those concerning the model of atmospheric turbulence. In addition, in practical calculations, some further simplifications are introduced the truth of which, being not self-evident, must be proved. This group of simplifications includes the assumption of a rigid wing, based on the experience of many years with sailplanes having 'old' stiffness parameters. However, this assumption must be examined concerning its applicability to modern high performance sailplanes made of plastics and having very high aspect ratios and wing spans.

The intent of the present paper is to analyse the correlation between the wing bending flexibility and the time history and peak values of loads (bending moments and shearing forces) due to gusts the intensity of which may determine the sailplane's ultimate strength. These loads are calculated by means of the discrete-gust approach.

In case of a rigid sailplane the time history of the loads due to gusts of high intensity shows that the load increase takes 0.25 to 1.0 second; in this connection the present paper takes into account merely the fundamental bending mode of the wing. The second mode natural frequency is 3 to 3.5 times higher, reaching 7 to 11 c/s for typical sailplanes.

Symbols

- b – wing span, m
- h – static margin
- l_a – mean aerodynamic chord of wing, m
- $l = \frac{l_a}{2}$ – reference length, m
- S – wing area, m²
- s_g – gust-gradient distance, m
- V – forward velocity of flight, m/s
- w_g – gust velocity, m/s
- w_o – max. gust velocity, m/s
- X – distance travelled, m

2. Discrete-Gust Models

Whenever the loads of an aircraft are to be calculated by the discrete-gust method the fundamental problem lies in chosing a proper distribution of air velocities (as to the magnitude and direction) over the gust-gradient distance.

An analysis of gust models adopted in a variety of sailplane design requirements shows that for sailplanes of orthodox design (as to both the arrangement and constructional parameters) one model of gust velocity distribution can be substituted by another model provided that the proper ratio of max. velocities is maintained, the final loads obtained being then nearly the same.

It is noteworthy that a common assumption for all the gust distributions is the vertical direction of gust velocity or perpendicular to the flight trajectory. The assumption of a defined gust model in a sailplane loading calculation is usually a rather arbitrary simplification; it pertains, in particular, to the 'pattern' of the gust velocity distribution, which has not been examined yet.

This paper makes use of 2 models of gust encounters, viz. the cosinusoidal gust (sometimes termed the 'one minus cosine' gust):

$$w_g = w_o \frac{1 - \cos\frac{\pi x}{s_g}}{2}$$

and the sinusoidal one:

$$w_g = w_o \sin \frac{\pi x}{2 s_g}$$

since they represent characteristic gust velocity distributions for the case of unidirectional ($w'_g(0) = 0$) and bidirectional ($w'_g(0) \neq 0$) gusts.

The cosinusoidal gust encounter can be considered a thermal model, while the sinusoidal gust velocity distribution represents rotor gusts (appearing under lee-wave conditions) being a boundary case of the wave motion.

A particular problem is what gust-gradient distances are to the associated with the two models of gust velocity distribution. This problem is more closely connected with the non-uniformity of the motion than with the methods of calculation of aerodynamic coefficients.

The assumption of a quasi-steady motion is justifiable for 'long' gusts causing a low rate of change of the sailplane angle of attack. The value of the non-dimensional distance to reach the maximum angle of attack increment after which the assumption of the quasi-steady flow is no longer justifiable as giving excessive values of maximum loads, depends on the parameter

$$\frac{\mu}{\frac{dC_z}{d\alpha}} = \frac{Q}{\frac{1}{2}\rho\, S.\, l_a \frac{dC_z}{d\alpha} g}$$

For sailplanes, for which the value of $\mu \div \frac{dC_z}{d\alpha}$ is low the error due to assumption of quasi-steady flow can be neglected whenever $s_g/l_a \geqslant 10$, then there is no need to take into account the Wagner effect for the gusts, whose gust-gradient distance exceeds 10 l_a.

The measurements show that the gust-gradient distance is closely connected with the gust maximum velocity. As the present paper is intended for analysis of loads for the purpose of determining the sailplane ultimatestrength, the analysis should cover high intensity gusts whose velocity w_o is 10 m/s at least. This velocity must be considered as being possible to be experienced even by a sailplane which is not admitted for cloud flying.

The results of measurements plotted by Donely [1] show that for all the gusts whose velocity is w_o is 10 m/s or more the lapse of time necessary for the attainment of the maximum load corresponds to a distance travelled X of 17 m or more.

For both the sinusoidal and cosinusoidal gust velocity distributions the maximum load appears before the gust velocity reaches its maximum; in this connection in the measurement data [1] all the gusts whose velocity w_o is 10 m/s or more have an equivalent wavelength of the sinusoidal gust

$\lambda > 4.17 = 68$ m

or an equivalent diameter of the thermal in the event of cosinusoidal gust velocity distribution

$d > 2.17 = 34$ m

In the two cases in question the gust-gradient distance is

$s_g > 10\, l_a$

so the assumption of steady flow for the entire range of gusts of $w_o \geqslant 10$ m/s and for the two gust velocity distributions under consideration is technically justifiable.

3. Methods of Analysis

The equations of motion for a rigid sailplane having two degrees of freedom and encountering a gust distribution $w_g = f(t)$ are as follows:

$$(-2\mu D^2 + C_{z\alpha} D)\, \bar{z} + 2\mu \bar{q} + C_{z\alpha}\, \overline{w}_o f(\bar{t}) = 0$$

$$(C_{m\dot{\alpha}} D^2 + C_{m\alpha} D)\, \bar{z} + (-i_B D^2 + C_{mq})\, \bar{q} + C_{m\alpha}\, \overline{w}_o f(\bar{t}) + (C_{m\dot{\alpha}} - C_{mq})\, \overline{w}_o f'(\bar{t}) = 0$$

where:

- $\bar{z} = \frac{z}{l}$ – vertical displacement in non-dimensional form
- $\overline{w}_o = \frac{w_o}{V}$ – velocity w_o in non-dimensional form
- $\bar{q} = q \frac{l}{V}$ – angular velocity about the lateral axis in non-dimensional form

$\mu = \frac{m}{\rho . S . l}$ – non-dimensional mass parameter

$i_B = \frac{I_y}{\rho . S\, l^3}$ – non-dimensional moment of inertia

$\bar{t} = \frac{t}{t^*}$ – non-dimensional time

$t^* = \frac{l}{V}$ – non-dimensional time unit

$D = \frac{d}{dt} = \frac{d}{dt} t^*$ – differential operator

$C_{z\alpha} = -\frac{dC_z}{d_\alpha};$

$C_{m\alpha} = \frac{dC_m}{d\alpha}; \; C_{mq} = \frac{dC_m}{d\bar{q}};$

$C_{m\dot{\alpha}} = \frac{dC_m}{d\left(\frac{\dot{\alpha} \cdot l}{V}\right)}$

If the third degree of freedom is taken into consideration in form of the wing natural vibration, the equations of motion of a rigid sailplane must become completed with aerodynamic forces and moments due to vibration and with a third equation added as resulting from the new degree of freedom.

In accordance with [2] the following system of equations is obtained:

$(-2\mu D^2 + C_{z\alpha} D)\, \bar{z} + \eta_1 r_1\, C_{z\alpha} D \bar{z}_1 +$
$2\mu \bar{q} + C_{z\alpha}\, \overline{w}_o f(\bar{t}) = 0$

$(C_{m\dot{\alpha}}\, D^2 + C_{m\alpha}\, D)\, \bar{z} + \eta\, {}_2 r_1\, C_{m\alpha\, wing}$
$D\bar{z}_1 + (-i_B D^2 + C_{mq})\, \bar{q} +$
$+ C_{m\alpha}\, \overline{w}_o f(\bar{t}) + (C_{m\dot{\alpha}} - C_{mq})\, \overline{w}_o f'(\bar{t}) = 0$

$r_1 C_{z\alpha}\, D\bar{z} + (-C\mu_1 D^2 - 2\mu_1\, \overline{\omega}_1^2 +$
$\eta_1 r_2 C_{z\alpha}\, D)\bar{z}_1 + r_1 C_{z\alpha}\, \overline{w}_o f(\bar{t}) = 0$

where:

$\bar{z}_1 = \frac{z_1}{l}$ – non-dimensional deflection of the wing tip (displacement with regard to the sailplane C. G.)

$\overline{\omega}_1 = \frac{\omega_1 l}{V}$ – non-dimensional angular frequency of the wing vibration

$\Phi_1\, /y/$ – normalized wing bending mode (for $z_1 = 1$)

$\mu_1 = \frac{2}{\rho\, Sl} \int_0^{b/2} \frac{dm}{dy}\, \Phi_1^2\, dy$

$r_1 = \frac{2}{S} \int_0^{b/2} l_y\, \Phi_1\, dy$

$r_2 = \frac{2}{S} \int_0^{b/2} l_y\, \Phi_1^2\, dy$

$\eta 1'\; \eta 2$ – coefficients representing the effect of unsteady flow on aerodynamic force and aerodynamic moment respectively

It should be noted that in the above system of equations $\bar{z}$ stands for a non-dimensional co-ordinate of motion of the sailplane C. G., the motion of any arbitrary point of the sailplane being described by the co-ordinate

$\bar{z}(y) = \bar{z} + \Phi_1(y)\, \bar{z}_1$

In order to determine the effect of wing flexibility on the wing loading the bending moment and shear force at the wing root have been compared with those calculated for a rigid wing. To illustrate the wing flexibility effect use has been made of the results of calculation of loads on a Zefir-2 sailplane, the main parameters being as follows:

- wing mass: from 140 kg (the Zefir 2, light, Q = 375 kg) to 205 kg (the Zefir 2, heavy, Q = 550 kg)
- frequency of the first bending mode of the wing: from 4 c/s to 1 c/s
- static margin: $h_{max} = 0.20$ to $h_{min} = 0.034$
- exciting load due to sine and cosine gusts; gust-gradient distance s_g = 15 min and 25 m.

4. Calculation Results

Making the stated assumptions, an analogue computer type Solartron HS7-2 has been used to compute the bending moments and shear forces at the wing root for all combinations of the following cases:

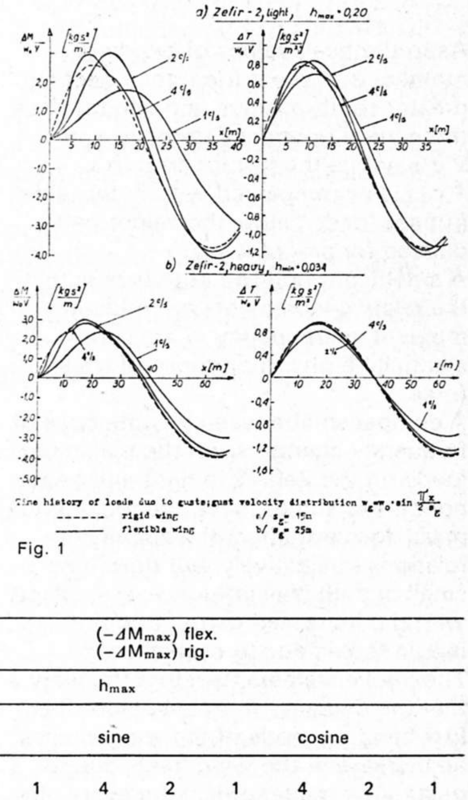
- the Zefir-2; light and heavy,
- static margin: minimum and maximum,
- gust velocity distribution: sinusoidal and cosinusoidal
- gust-gradient distance: s_g = 15 m and 25 m.

In all the 16 cases the calculations have been made for a rigid wing and for a flexible wing of variable natural frequency ν_1 = 1 c/s, 2 c/s and 4 c/s at the velocity V = 40 m/s.

The calculation results are shown in fig. 1 and 2.

The relative increments of peak values of bending moment resulting from the wing bending flexibility are presented in table 1, the shear force being shown in table 2.

It can be seen from the enclosed diagrams and tables that the loads on a flexible wing are substantially different from those on a rigid wing for frequencies of ν_1 = 1 c/s and 2 c/s, while at ν_1 = 4 c/s the differences are smaller. It may be concluded that for the higher frequencies the differences are not important in practice.



Time history of loads due to gusts; gust velocity distribution $w_g = w_o \cdot \sin \frac{\pi x}{2 s_g}$
------ rigid wing a/ s_g = 15m
——— flexible wing b/ s_g = 25m

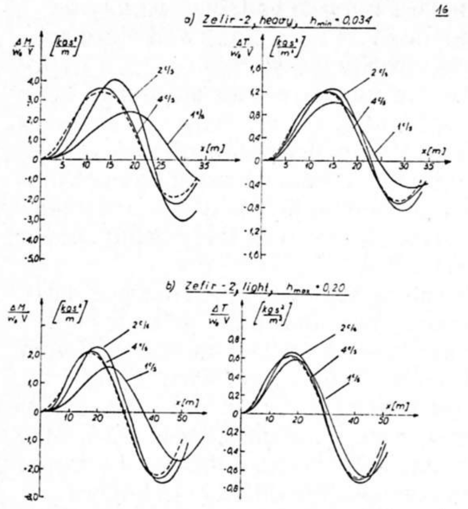
Fig. 1

Table 1. Bending Moment

		(ΔM_{max}) flex. / (ΔM_{max}) rig.						($-\Delta M_{max}$) flex. / ($-\Delta M_{max}$) rig.					
Static margin		h_{min}						h_{max}					
Gust velocity distribution		sine			cosine			sine			cosine		
Natural frequency c/s		4	2	1	4	2	1	4	2	1	4	2	1
Zefir-2	s_g = 15 m	1.05	1.08	0.75	1.08	1.10	0.61	1.14	1.15	0.66	1.10	1.10	0.59
light		1.0	0.96	0.65	1.11	1.36	0.28	1.03	1.10	0.82	1.12	1.30	0.41
	s_g = 25 m	1.01	1.04	0.71	1.03	1.10	0.79	1.02	1.17	0.81	1.04	1.11	0.77
		0.98	0.92	0.65	1.03	1.10	0.72	1.01	1.02	0.96	1.03	1.13	0.76
Zefir-2	s_g = 15 m	1.03	1.18	0.77	1.05	1.15	0.67	1.12	1.24	0.80	1.08	1.18	0.69
heavy		1.0	1.0	0.88	1.12	1.62	0.61	1.03	1.10	1.0	1.11	1.18	0.56
	s_g = 25 m	1.04	1.09	0.85	1.02	1.08	0.81	1.03	1.21	0.95	1.04	1.16	0.89
		1.0	0.95	0.75	1.02	1.06	0.93	1.01	1.04	1.08	1.04	1.15	1.0

Table 2. Shear Force

		(ΔT_{max}) flex. / (ΔT_{max}) rig.						($-\Delta T_{max}$) flex. / ($-\Delta T_{max}$) rig.					
Static margin		h_{min}						h_{max}					
Gust velocity distribution		sine			cosine			sine			cosine		
Natural frequency c/s		4	2	1	4	2	1	4	2	1	4	2	1
Zefir-2	s_g = 15 m	1.0	1.0	0.80	1.05	1.04	0.86	1.06	1.06	0.87	1.05	1.03	0.86
light		1.0	0.92	0.79	1.04	1.16	0.72	1.02	1.06	0.98	1.06	1.16	0.78
	s_g = 25 m	1.0	1.0	0.81	1.03	1.05	0.94	1.0	1.10	0.95	1.01	1.06	0.93
		0.98	0.91	0.68	1.0	1.04	0.92	1.0	1.0	1.0	1.02	1.08	0.97
Zefir-2	s_g = 15 m	1.0	1.06	0.84	1.03	1.07	0.87	1.05	1.10	0.89	1.04	1.05	0.85
heavy		1.0	0.96	0.89	1.06	1.27	0.67	1.01	1.05	1.03	1.06	1.23	0.75
	s_g = 25 m	1.0	1.0	0.86	1.02	1.06	0.96	1.0	1.11	0.98	1.02	1.08	0.96
		1.0	0.94	0.74	1.01	1.07	1.01	1.0	1.02	1.06	1.01	1.07	1.01



Time history of loads due to gusts; gust velocity distribution $w_g = \frac{w_0}{2}(1-\cos\frac{\pi x}{s_g})$

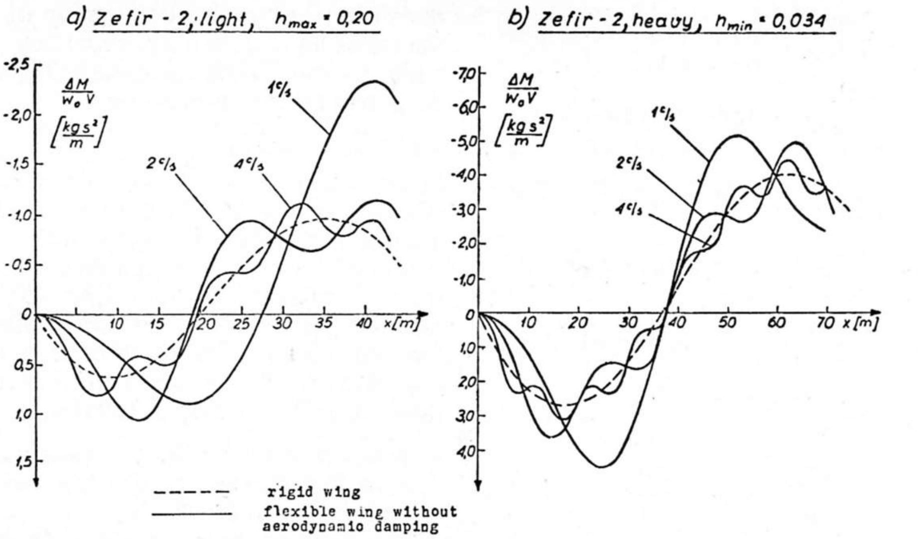
----- rigid wing

—— flexible wing

a/ s_g = 15m

b/ s_g = 25m

Fig. 2



Time history of wing bending moments due to gusts; gust velocity distribution $w_g = w_0 \sin\frac{x}{2 s_g}$

Fig. 3 a/ s_g = 15m b/ s_g = 25m

As to the peak values of bending moment and shear force they become greater (both positive and negative) as the natural frequency drops to ν_1 = 2 c/s, while the further drop to ν_1 = 1 c/s is accompanied with a decrease in peak loads below the values calculated for a rigid wing.

A significant point to note here is that the relative increments of bending moment vs frequency ν_1 are substantially higher than those of shear force.

A comparison between the influence of frequency change ν_1 on the maximum loads on the Zefir-2 in light and heavy conditions shows that a change of wing mass does not affect the above correlations qualitatively, but does have a small quantitative influence, associated with the increased contribution of inertia forces due to oscillation.

The above statements refute the view that the decrease in frequency of the first bending mode of the wing causes an increase in the wing loads due to gusts. As a matter of fact, for every gust, the maximum increment of wing loading corresponds to a different natural frequency, while the reduction in bending flexibility is accompanied at first by an increase in loads (with respect to those on a rigid wing) and then by a rapid decrease below the values corresponding to a rigid structure.

A significant point to note here is that the flexible wing when loaded does not oscillate at the wing natural frequency. This fact is due to intense aerodynamic damping, for which the non-dimensional coefficients $\bar{\xi}$ have been calculated in table 3 for the velocity V = 40 m/s. The heavily damped oscillations (superimposed on the main curve) are visible only for ν_1 = 4 c/s, while for ν_1 = 1 c/s and 2 c/s the damping is too intense.

Table 3

ν_1	c/s	1	2	4
$\bar{\omega}_1$		0.068	0.136	0.273
$\bar{\xi} = \frac{\eta_1 r_2 C_{z\alpha}}{4 \mu_1 \bar{\omega}_1}$	Zefir-2, light	1.0	0.425	0.185
	Zefir-2, heavy	0.65	0.29	0.125

In order to illustrate clearly the effect of aerodynamic damping of the wing bending oscillation on the load time history, the calculations have been repeated with the damping neglected. The results are presented in fig. 3.

A comparison of fig. 3 with fig. 1a + 1b shows that, if damping is neglected in simplified calculations, the results differ substantially from the true ones. Thus it seems pointless to derive from the above mentioned simplified calculations any rules of thumb for the use of aircraft designers; on the other hand the influence of aerodynamic damping is an important factor in the theoretical determination of the influence of wing bending flexibility on the wing maximum loads, so it is difficult to derive rules sufficiently accurate to be of practical help to the designer.

5. Concluding Remarks

1. For the gust conditions assumed in the present analysis, the influence of both the wing bending flexibility and mass on the maximum values (positive and negative) of loads is clearly visible; the wing flexibility can either increase or decrease the loads from those for a rigid wing. In addition the effect of wing bending flexibility on the wing loading depends on the characteristic of the sailplane short-period oscillations, this characteristic being dependent in turn on the sailplane static margin.
2. The range of variation of the first bending mode frequency extending from 4 to 1 c/s overlaps with some margin the typical frequencies of modern sailplanes as well as the anticipated characteristics of sailplanes made of laminated plastics having high aspect ratios and wing spans.
3. In a wing having a defined mass distribution the change in frequency of the first bending mode (i.e. a change in bending flexibility) from 4 to 1 c/s involves a change in maximum loads; in the determinant case of the least static margin the decrease in frequency from 4 to 2 c/s is accompanied in the main by an increase in the maximum loading (in particular if cosinusoidal gusts are encountered); a further decrease to 1 c/s results in reduction of loads below those calculated for a rigid wing.
4. The present analysis disproves the view that a reduction of frequency of the first bending mode of the wing is always accompanied by an increase in the wing loads resulting from gusts. As a matter of fact, for every gust, the maximum increment of wing loading corresponds to a different natural frequency, while a further decrease in the wing bending stiffness is accompanied by a rapid decrease in loads (below those calculated for a rigid wing). The incorrect view quoted above is probably based on the results of a simplified analysis, in which the influence of aerodynamic damping on the wing oscillation excited by gusts is neglected; the influence of aerodynamic damping on the wing loading is, on the contrary, of great importance.

References

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