# A Simplified Dynamic Speed Calculation Method for Flutter-Type Accident Investigation

Dr. J. Gedeon, Technical University Budapest, Hungary
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In order to find out the true cause of flutter-type sailplane accidents, a reasonably accurate knowledge of the airspeed is essential. For want of a speed recorder tape recourse to the flight barogram has to be made. Just before the accident the sink rate of the glider rarely remains constant for a sufficient long time for the static speed-sink relationship, given by the speed polar, to be directly applied. To treat such cases, a simplified numerical-graphical dynamic speed calculation method, accurate to some 8–10 km/h, has been developed. A second order approximation computer program is also being made.

## Notation

| | | |
|---|---|---|
| $c_y$ | lift coefficient | |
| $\delta c$ | vertical speed of ambient air | m sec$^{-1}$ |
| g | gravity acceleration | m sec$^{-2}$ |
| n = Y/G | normal load factor | |
| t | time | sec |
| $\Delta t = t - t_1$ | section time | sec |
| w | rate of sink | m sec$^{-1}$ |
| $\overline{w}$ | mean rate of sink for an element of time | m sec$^{-1}$ |
| G | flying weight | kp |
| H | height | m |
| $\delta H$ | difference of two subsequent height reading errors | m |
| S | wing area | m$^2$ |
| $T_1$ | time constant | sec |
| V | true air speed | m sec$^{-1}$ |
| $\overline{V}$ | equilibrium speed for sink $\overline{w}$ | m sec$^{-1}$ |
| $\Delta V = \overline{V} - V$ | | m sec$^{-1}$ |
| $\delta V$ | error in speed calculation | m sec$^{-1}$ |
| X | drag | kp |
| Y | lift | kp |
| | glide ratio | |
| | air density | kp m$^{-4}$ sec$^2$ |
| | glide angle | deg., rad. |

**Subscripts**

1 at beginning of element
2 at end of element

## 1. Statement of the Problem

Modern high-performance sailplanes are known for their good penetration, low internal noise level and moderate stick forces at high speeds. The resulting partial lack of speed sensation may mislead some pilots of moderate experience to try to regain some time lost in unnecessary circling by pushing the sail-plane to the limits. Should flutter or wing failure in bending occur in a high speed situation, then a reasonably true knowledge of the airspeed is essential for clearing up the cause of the accident, in order to prevent its recurrence.

Speed recorders are not carried on sailplanes except for special flight test work, and the investigators have no film or radar track records. Fortunately, a normal flight barogram may be a fairly reliable source of information for accidents in essentially straight flight, provided that the true speed polar of the sail-plane is known.

## 2. Speed Calculation for Short Flight Elements

In a practical high speed accident situation the case of at least 2–3 minutes of constant speed straight flying just before the event rarely if ever occurs, so the static speed-sink relationship given by the polar curve may not be directly applied. Let us take a look at a typical flight barogram like the one shown on the upper part of Fig. 1. There we have a high-time graph composed of elements being, to a good approximation, straight lines. If we are lucky, the high speed part of the graph may be preceded by an element long enough for equilibrium speed to be attained (see e.g. section 0 on Fig. 1). From this starting point we can set to work.

First, to each of the short straight line elements (element 1, 2, etc.) the mean sink speed $\overline{w}$ can be calculated. Then we may proceed as follows.

Supposing there is no forced speed correction, i.e. for $n \cong 1$, the dynamic equilibrium of forces acting on the sailplane is as given on Fig. 3. For the components parallel to the speed vector the force equation reads therefore:

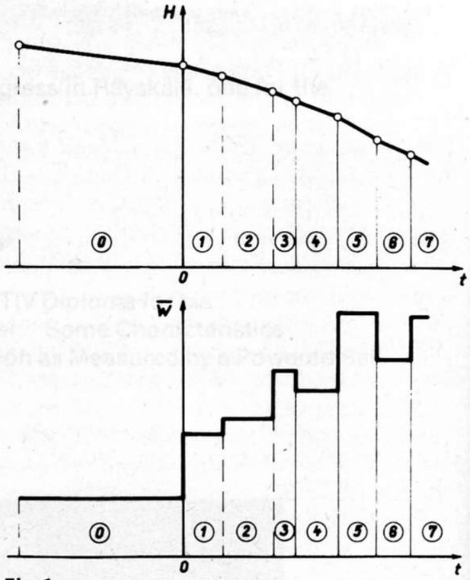

Fig. 1.
Typical flight barogram and sink rate curve.

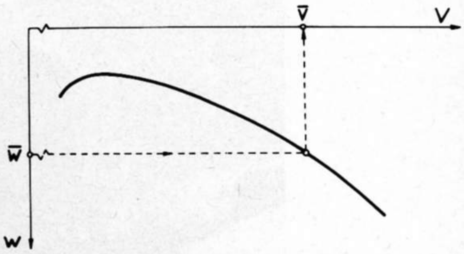

Fig. 2.
Speed polar showing equilibrium speed.

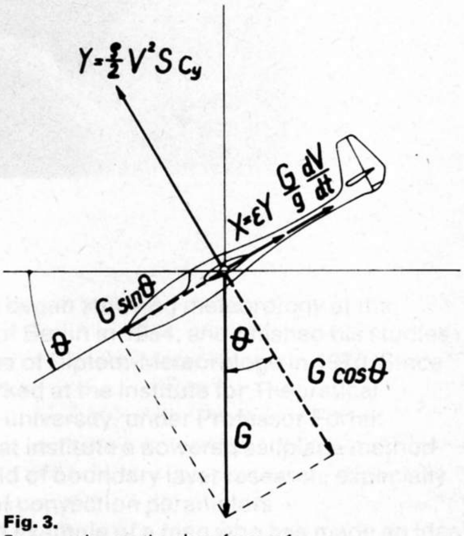

Fig. 3.
Forces acting on the plane for n = 1.

$$G \sin \Theta = \varepsilon G \cos \Theta + \frac{G}{g}\frac{dV}{dt} \qquad (1)$$

For the true air speed and sink we may write

$$\sin \Theta = \frac{w}{V} \qquad (2)$$

For not too steep glides $\cos\Theta \cong 1$, thus Equs. (1) and (2) are giving

$$\frac{w}{V} = \varepsilon + \frac{1}{g}\frac{dV}{dt}$$

or

$$\frac{dV}{dt} = g\left(\frac{w}{V} - \varepsilon\right) \quad (3)$$

Let us designate with $\bar{V}$ the equilibrium speed corresponding to the constant element sink $\bar{w}$. If we know (as e.g. at the end of element 0 on Fig. 1) the speed $V_1$ at the beginning of the element, for $t = t_1$ the equilibrium sink would be $w_1$ according to the polar curve. The corresponding glide ratio is:

$$\varepsilon_1 = \frac{w_1}{V_1} \quad (4)$$

From Equs. (3) and (4) the momentary value of the longitudinal acceleration is:

$$\frac{dV_1}{dt} = g\left(\frac{\bar{w}}{V_1} - \varepsilon_1\right) \quad (5)$$

If this acceleration remained constant then equilibrium speed would be reached in

$$T_1 = \frac{\bar{V} - V_1}{\frac{dV_1}{dt}} = \frac{1}{g}\,\frac{V_1\,(\bar{V} - V_1)}{\bar{w} - w_1} \quad (6)$$

For elements with moderate speed variation the initial value of the glide ratio may be taken for the whole element

$$t_1 \leqq t \leqq t_2 : \varepsilon \cong \varepsilon_1$$

simplifying the longitudinal equation of motion to

$$\frac{dV}{dt} = \frac{\bar{V} - V}{T_1} \quad (7)$$

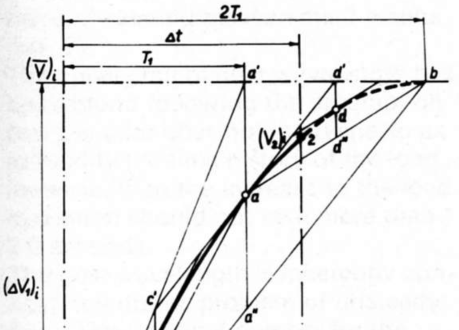


**Fig. 4.**
Section speed curve construction.

By introducing

$$\Delta V = \bar{V} - V : \frac{dV}{dt} = -\frac{d(\Delta V)}{dt};\ \Delta t = t - t_1$$

we get

$$-\frac{d(\Delta V)}{d(\Delta t)} = \frac{\Delta V}{T_1} \quad (8)$$

The solution of this differential equation for

$$t = t_1 : \Delta V = \Delta V_1$$

as initial conditions is

$$\Delta V = \Delta V_1\, e^{-\frac{\Delta t}{T_1}} \quad (9)$$

For the short elements we are dealing with this may be substituted with sufficient accuracy by

$$\Delta V \cong \Delta V_1 \left[\frac{2T_1 - \Delta t}{2T_1}\right]^2 \quad (9a)$$

Practical computation is continued as follows (see Fig. 4):
Supposed we know the initial speed of element I, from the foregoing element as $(V_1)_i = (V_2)_{i-1}$

in addition $\bar{w}$ and – from the polar – $\bar{V}$. The value of $w_1$ can also be read from

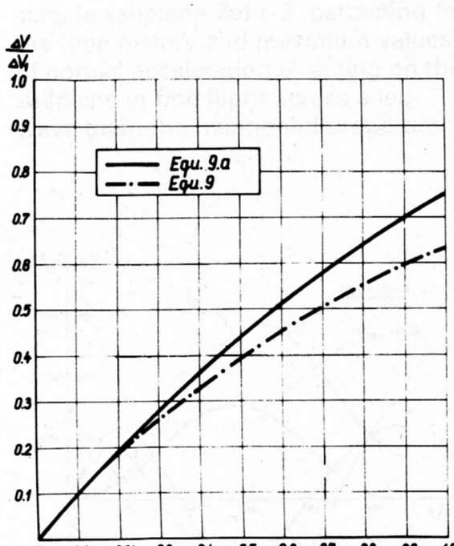


**Fig. 5.**
Exponential vs. parabolic speed transition.

the polar curve so $T_1$ can be found from Equ. (6). The parabolic arc of the speed curve for the element may be constructed by some convenient method like that shown on Fig. 4. At the end of the element we get $(V_2)_i = (V_1)_{i+1}$ leading to the next element.

## 3. Probable Error Calculation

For an indirect speed calculation r method to be trustworthy some form of error estimation procedure is essential. The results of the foregoing speed determination procedure may be inaccurate because of errors in the speed polar, up- and downdrafts, errors in the barogram reading, (systematical) calculation inaccuracies, etc.
Speed polar errors may be directly assessed either from the known confidence limits or, as shown on the numerical example on Fig. 6, by working with two different polars.

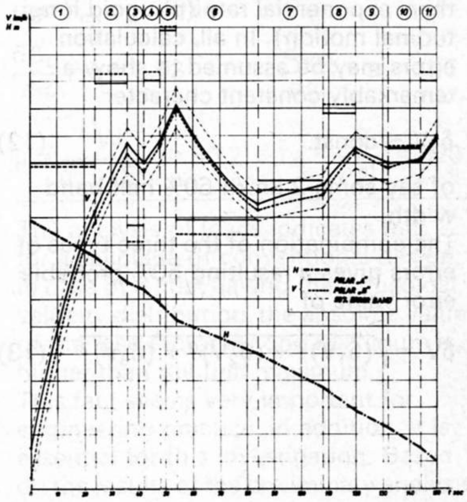

**Fig. 6.**
Numerical example.

### Lift/Sink Error

If there is an unknown vertical air movement of speed $\delta c$ over the whole length of the element then the speed error due to this, at the end of the element, is

$$\delta_1 V = \delta c\,\frac{dV_2}{dw} \quad (10a)$$

For the longer element there is a fair possibility of the vertical air motion not extending over the whole length of the section but only over say some 800 m of it. In this case we may calculate (in km/h) with

$$\delta_1 V = \delta c\,\frac{dV_2}{dw}\,\frac{2900}{V_2\,\Delta t} \quad (10b)$$

### Barogram Reading Error

As it may be easily shown, the fundamental source of this type of error is the difference between two subsequent readings $\delta H$. This is accounting for an error

$$\delta w = \frac{\delta H}{\Delta t}$$

in the mean sink calculation. The resulting speed error at the end of the section may be written as

$$\delta_2 V = \frac{\delta H}{\Delta t}\,\frac{dV_2}{dw}\left[1 - \left(1 - \frac{\Delta t}{2T_1}\right)^2\right] \quad (11)$$

### Calculation Errors

Up to now, no proper justification for substituting the parabolic speed transition law (Equ. 9a) for the exponential one (Equ. 9) has been given. In this respect we may refer to Fig. 5 showing the difference in the two respective speed curves. The parabolic law is giving a more rapid trend to attain equilibrium speed than does the exponential one. This is not a source of, but a correction for, an error. As it is known from longitudinal stability analysis, equilibrium speeds are approached always at a faster than exponential rate (phugoid longitudinal motion). In all, calculation errors may be assumed to show a remarkably constant character

$$\delta_3 V \cong \text{const.} \qquad (12)$$

of say some 2 km/h 50% half band width.
The combination of the three types of errors gives a resulting 50% probable error value of

$$\delta V = \sqrt{(\delta_1 V)^2 + (\delta_2 V)^2 + (\delta_3 V)^2} \qquad (13)$$

In practice this may amount to not more than 8–10 km/h. Results of a realistic numerical example are shown on Fig. 6.

### 4. Refinement Possibilities

In its present form, the preceding method is still deficient in some respects. There is no possibility of accounting for normal acceleration effects and the sink "jumps" at section boundaries are also unnatural. It is planned therefore, in continuing the investigation, to supplement it by the equation of motion for normal acceleration and to smooth the barogram curve as shown on Fig. 7. In its extended form the problem will be somewhat similar to the transient dolphin-flight calculation case, so it may similarly well respond to finite time element calculation methods. Work is being done in this direction and the program will be made accessible to all in need of it by OSTIV.

### Acknowledgements

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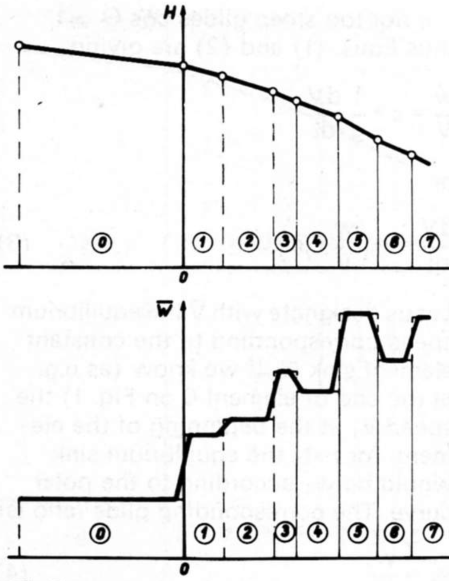


**Fig. 7.**
Smoothing of sink rate curve corners.