Shapes for the tips of lightly-loaded propeller blades

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Summary
The considerations of this paper are relevant to maximising the efficiency of lightly-loaded propellers, particularly such as are used on man-powered aircraft. They are also relevant, in a more qualitative fashion, to more highly-loaded propellers, (e.g. for motorgliders) where noise reduction is important.

Professor E.E. Larrabee has observed that there is an error in Glauert's analysis of the effect of a finite number of propeller blades (Durand, Vol. IV). When the appropriate correction is made, it is found that special attention has to be paid to the developed planform shape of the blade tips if the local axial and circumferential induced velocity factors are not to become infinite or indefinite.

It is shown that if the tip shape is "sharper" than a parabola, these factors become zero at the tip whilst a tip of suitably rounded form but "blunter" than a parabola can lead to finite values of the factors. The former condition seems preferable both on grounds of improved efficiency and diminished noise.

Introduction
The application of vortex theory to a propeller with a large number of blades shows that the expressions for the axial and circumferential induced velocity factors are, respectively, as follows:

$$\frac{\alpha}{1+\alpha} = \frac{C_L\sigma}{4}\,\frac{\cos(\Phi+\gamma)}{\cos\gamma\,\sin^2\Phi}, \qquad (1)$$

$$\frac{\alpha'}{1-\alpha'} = \frac{C_L\sigma}{4}\,\frac{\sin(\Phi+\gamma)}{\cos\gamma\,\sin\Phi\,\cos\Phi}. \qquad (2)$$

Here, $\gamma = \tan^{-1}(C_D/C_L)$ and hence these expressions correspond to equations 3.2 and 3.3, p. 236, Ref. 1. (See also the list of symbols and Fig. 1).
Prandtl considered the effect of a finite number of blades. Since there is no flow through the vortex sheets trailing behind the blades in the slipstream, he suggested, as an approximate analogy, that the radial flow near the edges of the vortex sheets would be similar to the two-dimensional flow about a series of semi-infinite flat plates moving through the fluid. An analysis of this flow leads to the factor F, where

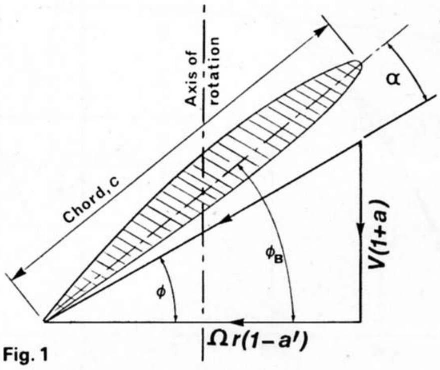


Fig. 1

Conditions at a blade element at radius r.

$$F = (2/\pi)\cos^{-1}[\exp(-f)] \qquad (3)$$

and $$f = (B/2\lambda)\left(\sqrt{1+\lambda^2}\right)(1-\zeta). \qquad (4)$$

In Glauert's words [1]: ". . . F is a reduction factor which must be applied to the momentum equation for the flow at radius r, since it represents the fact that only a fraction F of the air between the successive vortex sheets of the slipstream receives the full effect of the motion of these sheets".
It then follows that equations (1) and (2) must be modified by multiplying their right-hand sides by $(1/F)$. This seems obvious, in the sense that αV and $\alpha'\Omega r$ represent the axial and circumferential velocity components induced at the blade element by the trailing vortex system and will therefore be greater than the corresponding average values. In detail, this result may be obtained by following the analysis of Ref. 1. Equation (1) then becomes

$$\frac{\alpha}{1+\alpha} = \frac{C_L\sigma}{4}\,\frac{\cos(\Phi+\gamma)}{\cos\gamma\,\sin^2\Phi}\,\frac{1}{F} \qquad (5)$$

and similarly for equation (2).

As Professor E.E. Larrabee has observed,[2] equations 5.5 on p. 268 of Ref. 1 are in error because, as printed, F appears in the numerator, not the denominator.

Now, as $\zeta \to 1$ near the blade tip, $f \to 0$ and hence $F \to 0$. So, if all the other quantities on the right-hand side of equation (5) are finite, α will tend to infinity as $\zeta \to 1$, as also will α'. This is analogous to the induced downwash at a wingtip tending to infinity if a vortex of finite strength is shed from the tip.
Even if C_L and/or σ are arranged to be zero at the tip, α and α' may well be indefinite. We therefore enquire whether σ/F can be arranged to tend to zero, or some finite value, as $\zeta \to 1$.

Analysis
Now f can be written $k(1-\zeta)$, where k is a function of B and λ, and is therefore a constant for a given propeller operating at a fixed advance ratio.

For small $(1-\zeta)$,
$$\exp(-f) \doteqdot 1-k(1-\zeta). \qquad (6)$$

Also, $\cos^{-1}\kappa \to \sqrt{1-\kappa^2}$ as $\kappa \to 1$, so

$$\cos^{-1}[\exp(-f)] \to \sqrt{2k(1-\zeta)} \text{ as } \zeta \to 1.$$

Finally, $F \to \frac{2}{\pi}\sqrt{2k(1-\zeta)}$
as $\zeta \to 1$. (7)

Now, putting $c/R = \chi$, σ becomes

$$\sigma = (B/2\pi)(\chi/\zeta). \qquad (8)$$

So $$\sigma/F \to \frac{B}{4\sqrt{2}\,k}\,\frac{\chi}{\zeta\sqrt{1-\zeta}} \qquad (9)$$

as $\zeta \to 1$.

If σ/F is to tend to zero as $\zeta \to 1$, then $\chi/\sqrt{1-\zeta}$ must tend to zero as $\zeta \to 1$.
One possibility is to make

$$\chi = \kappa(1-\zeta)^n \qquad (10)$$

where κ is a constant and $n > \frac{1}{2}$. The "limiting" tip shape, (i.e. with $n = \frac{1}{2}$) is therefore parabolic. For a given blade chord at, say, $\zeta = 0.9$, any tip for which $n > \frac{1}{2}$ will lie inside the limiting parabola.
The slope of the tip profile, $d\chi/d\zeta$, is proportional to $(1-\zeta)^{n-1}$. So, as $\zeta \to 1$, the tip slope will tend to $(-\infty)$ if $n < 1$ (i.e. the tip will be blunt) and to zero if $n > 1$ (i.e. the tip will be pointed). It therefore seems that, on practical grounds $1 > n > \frac{1}{2}$.

The parabolic tip shape ($n = \frac{1}{2}$) leads to a finite value of σ/F at $\zeta = 1.0$. For $n > \frac{1}{2}$, $\sigma/F = 0$ at $\zeta = 1.0$.

An alternative possibility is to allow σ/F to attain some finite value at the tip. If the tip is to be blunt, the conditions to be fulfilled are:

$$\frac{\chi}{\zeta\sqrt{1-\zeta}} \rightarrow \tau, \text{ where } \tau \text{ has some finite value and } d\chi/d\zeta \rightarrow -\infty,$$

$$\chi = \kappa(1-\zeta)^{\frac{1}{2}}\, g(\zeta), \qquad (11)$$

where $g(\zeta)$ is some function of ζ, then the conditions will be met if $g(\zeta)$ and $g'(\zeta)$ are finite at $\zeta = 1$. A simple relationship which meets this requirement is

$$g(\zeta) = \zeta^m, \qquad (12)$$

The case $m = 0$ then corresponds to the previous limiting parabola. If $m > 0$, the tip is "blunter" than the parabolic shape (see Fig. 2). The case $m=1$ corresponds to a constant value of σ/F over the tip region, whilst if $m > 1.0$, σ/F increases towards the tip (Fig. 3).

Remarks

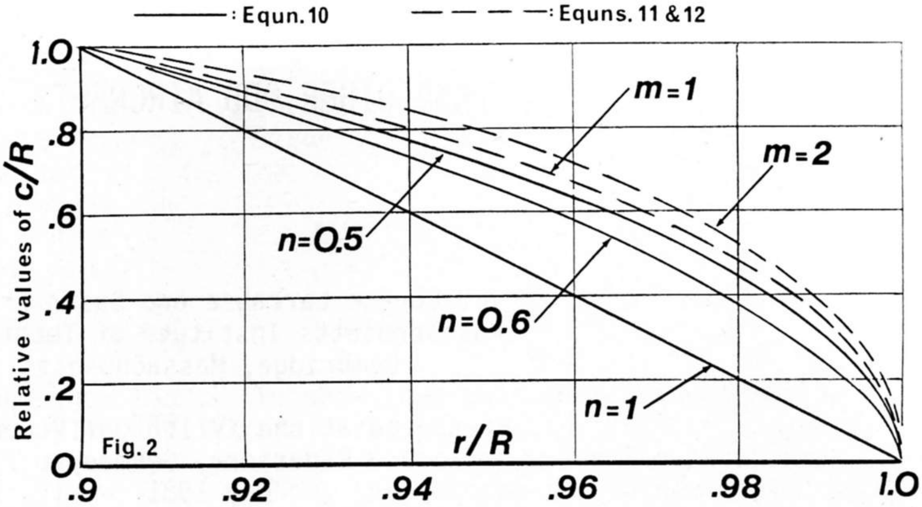
If the blade sections have the same lift/drag ratio at all values of ζ, the optimum distribution of circulation along the blade is such that the circulation, and hence the factors α and α', fall to zero at the tip. In the present context, the Reynolds number may vary appreciably along the blade, with corresponding variations in the lift/drag ratio. The simple analysis of Ref. 1 then no longer applies. If the blade chord near the tip is small, its lift/drag ratio is likely to be poor compared with that prevailing elsewhere and, qualitatively, it seems even more desirable to reduce the loading in this region.

With more highly-loaded propellers (on motor-gliders, for example, and certainly on larger powered aircraft), it seems desirable to reduce α and α' to zero at the blade tips so as to reduce the rate of shear at the slipstream boundary and thus to diminish the noise.

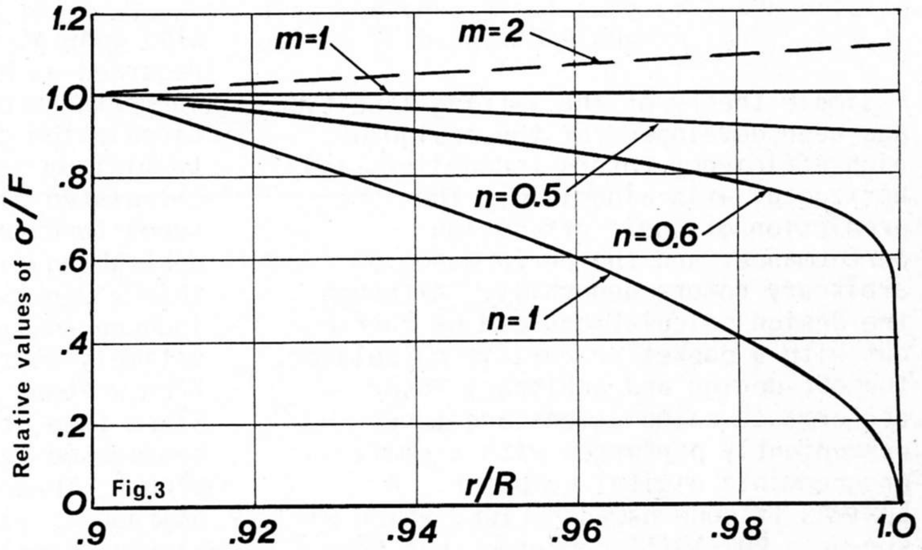
The axial and circumferential induced velocity factors will only be zero at the slipstream boundary at all advance ratios if σ/F becomes zero at the blade tip. These factors will, in general, have finite values at the boundary if σ/F tends to some finite value at the blade tip. The characteristics of the propeller will be calculable, but the resulting thrust distribution will probably be non-optimum.

Conclusion

In the interests of efficiency and, in some applications, noise reduction, it is usually desirable that the thrust



Some possible tip shapes. The values of c/R are scaled so as to be the same at $r/R = 0.9$.



Relative values of σ/F near the tips of propellers of various shapes.

grading and hence the induced velocity factors α and α' should diminish to zero at the blade tip. This can be achieved for all working conditions by ensuring that σ/F becomes zero at the tip.

A simple tip shape of rounded form which satisfies this condition is

$$\frac{c}{R} = \kappa\left(1 - \frac{r}{R}\right)^n \qquad (13)$$

where $1 > n > \frac{1}{2}$. The conditions $n=1$ and $n=\frac{1}{2}$ are excluded: the former corresponds to a triangular tip, the latter to a finite value of σ/F. It should be possible to choose values of κ and n so that the tip shape fairs smoothly into the blade shape inboard of say $r/R = 0.9$. These shapes are all "sharper" than the parabola corresponding to $n=\frac{1}{2}$.

Tip shapes corresponding to

$$\frac{c}{R} = \kappa\left(1 - \frac{r}{R}\right)^{\frac{1}{2}}\left(\frac{r}{R}\right)^m$$

where $0 < m$, lead to finite values of σ/F at the tip, except when the aerodynamic terms in the numerators of equations (1) and (2) are zero. These shapes are "blunter" than parabolic.

It would seem that shapes corresponding to equation (13) are generally to be preferred.

References

1. "Aerodynamic Theory" (Ed.: W.F. Durand), Div. L, "Airplane Propellers" by H. Glauert.
2. Larrabee, E.E. Private Communication.

List of symbols

α	Axial induced velocity factor
α'	Circumferential induced velocity factor
B	Number of blades
C_L	Local blade lift coefficient
C_D	Local blade drag coefficient
c	Local blade chord
F	Prandtl's factor (see equation (3))
f	See equation (4)
κ	A constant
m	An index (equation (12))
n	An index (equation (10))
r	Local blade radius
R	Tip radius
V	Forward speed of the propeller
Φ	Flow direction at a blade element (see Fig. 1)
γ	$\tan^{-1}(C_D/C_L)$
κ	A constant (equation (10))
λ	$V/\Omega R$
σ	Local solidity, $Bc/2\pi r$
τ	A constant
ζ	Dimensionless radius, r/R
χ	Dimensionless blade chord, c/R
Ω	Angular velocity of the propeller.