A simple model of dynamic energy exchange between a sailplane and vertical air currents

Justin Sandauer, presented at the XVIIth OSTIV Congress, Paderborn, Germany (1981)

In the paper "Energy Exchange Between a Sailplane and Moving Air Masses..." by W. Gorisch presented on the XV. OSTIV Congress, the principles of optimal choice of airspeed and normal accelerations for a non-stationary dolphin-mode flight were given. Also the author's paper "Some Problems of the Dolphin-Mode Flight Technique" presented at the XVI. OSTIV Congress concerned, in part, the same problem.

The use of dynamic speed changes when flying through areas of vertical currents is aimed at increase of power transferred from the atmosphere to the sailplane. This power may be determined on the basis of a fundamental relation of mechanics stating that the derivative with respect to time of the total mechanical energy of a rigid body is given by the scalar product of the force affecting a body and its velocity inertial co-ordinate system.

The formula for the power transferred from the atmosphere to the sailplane flying through a vertical air current of constant velocity w_{atm} was given by Gorisch [1]:

$$E' = \frac{dE}{dt} = (\vec{L} + \vec{D})\ (\vec{V} + \vec{w}_{atm}) = \qquad$$
$$= L \cdot w_{atm} \cdot \cos\varphi - D \cdot V \qquad (1)$$

where φ is the angle between lift L and air velocity vector w_{atm}.

The first term of the right side of this equation is the power transferred from the atmosphere to an ideal sailplane (with zero-drag) and the second term is the power lost due to the aerodynamic drag.

Since both terms depend on the aerodynamic force which can be represented by an appropriate g-number, the rate of energy transfer must depend strongly on the g-loading; thus an optimal value of load factor n_{opt} corresponding with the maximum of power transferred from the atmosphere to the sailplane can be calculated. The higher the gliding ratio of the sailplane at a given airspeed, the lower is the contribution of the equation's second term and the higher is the value of n_{opt}. The same effect has also an increase of the effective intensity of the updraft $w_{atm} \cdot \cos\varphi$, ← which influences the first term of formula (1) and results in the increase of the optimum value of the load factor n_{opt}. It seems however, that this simple—from the point of view of mechanics—reasoning is not readily understandable by means of widely known physical terms such as potential and kinetic energy. At the same time it is obvious that the usefulness of dynamic controlling of the sailplane in dolphin-mode flight through vertical air currents for additional energy gain must be explained also to pilots having poor mathematical background. It is possible only on the basis of simple and easily understandable physical models.

For better presentation of the problem let us analyse the case of an ideal sailplane in a level flight at the airspeed V and encountering an updraft having a velocity

w_{atm}. Thus ($\varphi = 0$):

$$E' = L \cdot w_{atm} \qquad (2)$$

and therefore the power transferred from the atmosphere is proportional to the lift L.

If the pilot does not perform a dynamic pull-out then:

$$L = m \cdot g \quad \text{and} \quad E' = m \cdot g \cdot w_{atm}$$

and the power absorbed from the atmosphere transfers in time Δt into the potential energy increment:

$$\Delta E_{pot} = m \cdot g \cdot w_{atm} \cdot \Delta t = m \cdot g \cdot \Delta h \qquad (3)$$

which is physically evident.

For precision it must be mentioned that a "step" increment of the sailplane's kinetic energy

$$\Delta E_{kin} = \frac{m \cdot w_{atm}^2}{2}$$

proportional to the square of the velocity of transportation w_{atm} occurs at the moment of entry into the updraft. Let us now examine the pull-out manœuvre. In a co-ordinate system moving together with the vertical air current (Fig. 1) a sudden change of angle of attack induces a lift increment ΔL and consequently a curvature of the flight path. In time Δt the sailplane moves from the point 1 to the point 2.

Following the formula (2) the additional power absorbed from the atmosphere

$$E' = \Delta L \cdot w_{atm} \cdot \cos\varphi \qquad (4)$$

and the additional energy increment at the point 2 (due to dynamic manœuvre):

$$\Delta E = \Delta L \cdot w_{atm} \cdot \cos\varphi_{mean} \cdot \Delta t \qquad (5)$$

Let us now try to find the physical form of this additional energy.

The potential energy increment $m \cdot g \cdot \Delta z$ due to the fact that point 2 lies higher then point 1 is of no importance because it is compensated by equivalent drop of kinetic energy due to the loss of sailplane's airspeed; in such a case we observe only a change of the form of energy and consequently no gain of total energy occurs. Thus we can neglect this potential energy increment assuming simultaneously that the airspeed on the way from 1 to 2 remains constant.

Let us now seek the additional energy increase, evaluated by means of formula (5), in form of kinetic energy. For this purpose we must resolve the airspeed at the point 2 into horizontal and vertical components (Fig. 1). Taking into account the transportation velocity of co-ordinate system w_{atm} the total kinetic energy of the sailplane at the point 2:

$$E_{kin} = \frac{m}{2}(V \cdot \cos\varphi)^2 + \frac{m}{2}(V \cdot \sin\varphi + w_{atm})^2 =$$
$$= \frac{m \cdot V^2}{2} + \frac{m \cdot w_{atm}^2}{2} + m \cdot w_{atm} \cdot V \sin\varphi \qquad (6)$$

The first two terms of the right side of this equation represent the kinetic energy of the sailplane in a straight flight i.e. at

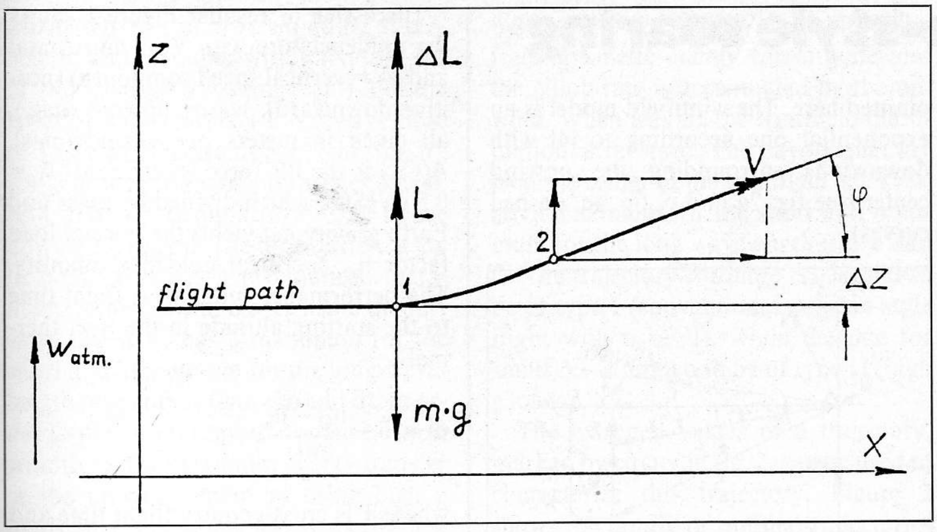

Fig. 1 Flight path of a sailplane in a co-ordinate system moving with velocity w_{atm}.

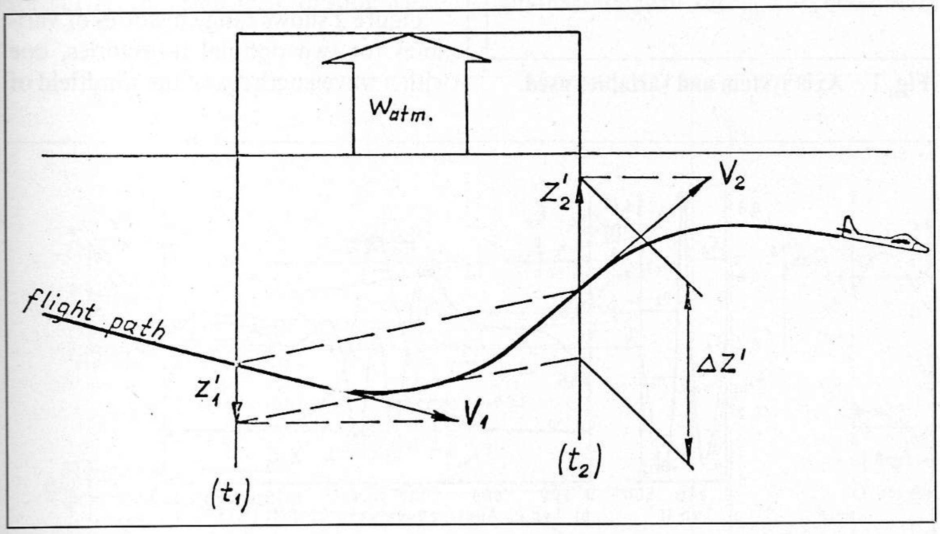

Fig. 2 Flight path of a sailplane performing a dynamic manœuvre in an area of constant vertical velocity.

the point 1, while the third term represents an amount of additional energy due to the curvature of the flight path in a velocity of transportation field. The formula (6) shows clearly that the necessary conditions for the occurrence of the additional kinetic energy are two – flight path curvature angle φ and velocity of transportation w_{atm}.

Now it must still be proved that the power required for the energy gain is equal to the power absorbed from the atmosphere according to the formula (4):

$$E' = \frac{d}{dt}(m \cdot w_{atm} \cdot V \sin \varphi) = m \cdot w_{atm} \cdot V \cdot \frac{d\varphi}{dt} \cos \varphi$$

$$= \Delta L \cdot w_{atm} \cdot \cos \varphi$$

since $mV \cdot \frac{d\varphi}{dt} = \Delta L$

Let us now sum up the above considerations as follows: the positive or negative energy exchange between the atmosphere and a sailplane resulting from dynamic longitudinal manœuvres during flight through vertical air currents can be easily explained to a pilot as an additional amount (positive or negative) of kinetic energy due to the fact that the vertical component of sailplane's velocity with respect to the earth is a sum of the vertical component of sailplane's airspeed and the velocity of transportation (velocity of the air current); the kinetic energy of the resultant motion is proportional to the square of this sum and therefore is higher than the sum of the energy of both separate motions.

In the same manner can be explained a formula given by Gorisch, concerning the sailplane's energy gain in a flight through an area of constant vertical velocity w_{atm} (Fig. 2):

$$\Delta E = m \cdot w_{atm}\ \Delta Z' + m \cdot g \cdot (w_{atm} - w_{mean}) \cdot \Delta t \quad (7)$$

where the first term of the right side is the additional energy increment due to the dynamic manœuvre.

Also conclusions drawn by the author (lit. 2) concerning optimization of non-stationary dolphin-mode flight can be easily understood when this simple model of dynamic energy exchange is used.

References

1. *W. Gorisch:* Energy Echange Between a Sailplane. OSTIV Publication XIV, Rayskälä, 1976

2. *J. Sandauer:* Some Problems of the Dolphin-Mode Flight Technique. OSTIV Publication XV, Chateauroux 1978

Zusammenfassung:

Ein einfaches Modell zur Wechselwirkung von Bewegungsenergie zwischen einem Segelflugzeug und vertikaler Luftströmung

Der positive oder negative Energieaustausch zwischen einem Segelflugzeug und der umgebenden Luftströmung erfolgt im wesentlichen bei Bewegungsvorgängen entlang der Längsachse des Flugzeuges durch Auswirkung vertikaler Bewegungskomponenten der Luft. Dies kann relativ einfach verstanden werden als ein positiver oder negativer Zusatzbetrag zur kinetischen Energie des Flugzeuges, wenn man annimmt, dass die vertikale Komponente der Geschwindigkeit des Flugzeuges relativ zur Erde die Summe von Vertikalkomponente der Flugzeugeigengeschwindigkeit und der Geschwindigkeit der Luft ist.

Die kinetische Energie der resultierenden Bewegung ist dann proportional zum Quadrat dieser Summe und daher grösser als die Summe der Energie beider getrennt verlaufenden Bewegungen.