Observations of waves in a wide range of conditions

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Presented at the XVII. OSTIV-Congress, Paderborn (F.R.G.), 1981

Introduction: Mathematical models of lee waves have advanced considerably since the early two layer model described by Scorer (1949). Although the early models did not include the effect of the stratosphere they provided a useful guide to the conditions required for waves in the troposphere. Corby and Sawyer (1958) used a four layer model to study the effect of the stratosphere; they noted that lee waves could still occur when Scorer's criteria were not satisfied but that the waves would die out downstream. Since then the increase in computing power has made possible the development of multi-layered time-dependant models for the study of evolving wave patterns. However the growing complexity of numerical models has not so far led to an equivalent improvement in the day to day forecasting of lee wave activity. Most forecasters are still obliged to rely on graphical methods such as those proposed by Casswell (1966) or Lester (1975).

It therefore seemed worthwhile to make a survey of the range of conditions under which lee wave soaring had occurred and to examine some simple lee wave models which could be used on a programmable pocket calculator.

Sailplane pilots were asked to report occasions when they were able to make climbs of more than 3 km in wave over the British Isles. Several hundred reports were examined using synoptic data from the Daily Weather Report and the Daily Aerological Record. The location of gliding sites from which the wave flights began is shown in Fig 1 together with the positions of radiosonde stations used in the study.

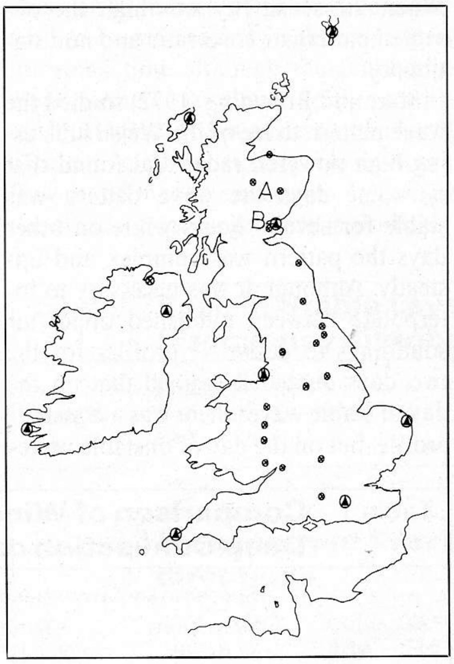
It was often necessary to use several upper air soundings to obtain a representative profile of wind speed and temperature over the areas of wave flights. Even when a sounding was available very close to the climb it was usually necessary to smooth out minor irregularities in order to reduce the number of layers for which Scorer's parameter 1^2 was calculated.

The need for smoothing was reported by Danielsen and Bleck (1970) although Sawyer (1960) had observed that irregularities in the 1^2 profile were not reflected in the streamlines calculated from a multilayered model.

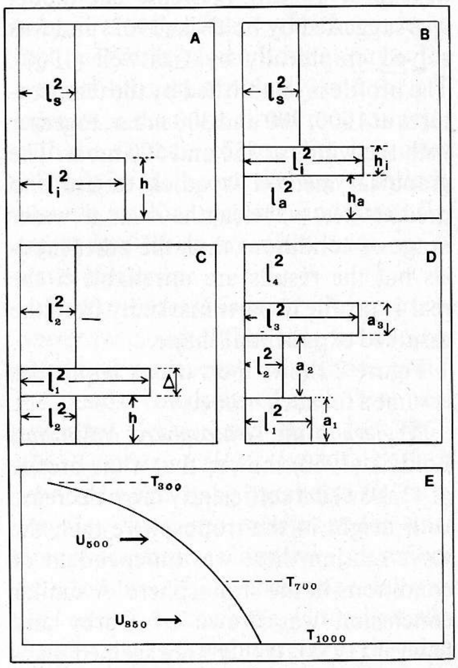
It was hoped that errors introduced by interpolation and smoothing would be less serious than those caused by unrecorded changes in wind velocity.

Use of Models: In order to keep within the very limited capacity of a pocket calculator it was necessary to use only the simplest models. These were:

(a) Scorer's two layer model (1949) which requires only three factors to determine if waves are possible and if so what the wavelength is. These factors are 1_i^2 and 1_s^2, the Scorer parameters for the inferior and superior layers respectively, and h the height of the interface between the two layers. The criterion for lee waves is that $1_i^2 - 1_s^2 < \pi^2/4h^2$. Even when the stratosphere was neglected the atmosphere could seldom be represented by just two layers.



1. Locations of gliding sites from which wave flights began, and radiosonde stations used in survey. Letters "A" and "B" apply to areas referred to in text.



2. Five simple lee wave models. Vertical profiles of 1^2 parameter.
A: Scorers 2 layer model.
B: Wallingtons 3 layer model
C: Pearce & White 3 layer model
D: Corby & Sawyer 4 layer model
E: Foldvik exponential profile (used by Casswell).

(b) A three layer model suggested by Wallington (1955). This employs Scorer's method, but with the addition of an adiabatic layer where 1^2 is zero underneath the stable layer. This model was found to give a good representation of tropospheric conditions on many wave days.

(c) A three layer model described by Pearce and White (1967) which is similar to Wallingtons model in having the stable layer in the middle but differs in that the value of 1^2 above and below must be adjusted to be equal.

(d) A four layer model described by Corby and Sawyer (1958). This included a stable layer in the stratosphere. It took longer to run but was useful on days when the tropospheric models failed to predict lee waves.

(e) A model in which the value of 1^2

decreased exponentially from the surface up to the level of maximum wind, instead of altering in steps. The model was suggested by Foldvik (1962) and was solved graphically by Casswell (1966). The profile is established by the temperatures at 1000, 700 and 300 mbar, together with the winds at 850 and 500 mbar. The graphical method is quick to use and predicts lee wavelengths over a wider range of conditions than the other models but the results are unreliable if the real l^2 profile departs markedly from the assumed exponential shape.

Figure 2 shows the various l^2 profiles assumed for each model.

Neglect of the stratosphere: Palm and Foldvik (1960) showed that if the profile of l^2 showed a sufficiently large decrease with height in the troposphere then the wave motion there was independant of conditions in the stratosphere. A similar conclusion was drawn by Corby and Sawyer (1958). It therefore seemed justifiable to ignore stratospheric conditions when calculating lee wavelengths, provided that Scorer's criteria were amply fulfilled. A comparison of calculated and measured wavelengths showed that no major errors were introduced on days when satellite pictures showed well developed wave trains.

The problem of wave amplitudes and vertical velocities: It is theoretically possible to calculate streamlines, wave amplitudes and vertical velocities from simple lee wave models but it is doubtful if the results have much validity in practical problems. The simple models are based on linear equations and assume that the wave amplitude is proportional to the forcing effect of a ridge whose size and shape can be specified.

Corby and Wallington (1956) showed that wave amplitude is extremely sensitive to variations of the l^2 profile and the dimensions of the ridge. Smith (1976) pointed out that linear lee wave theory may badly under estimate the true amplitude because of the effects of strong nonlinearity in the governing equations. Peltier and Clark (1979) suggested that waves might be trapped and subsequently amplified beneath their own level of supercritical steepening; although this idea was disputed by Lilley and Klemp (1980) they agreed that some unexplained amplification mechanism comes into play in the presence of wave breakdown. Brown and Stewartson (1980) pointed out that when linear theory is valid the critical layer is a wave absorber, but on longer time scales the critical layer starts to reflect wave energy.

It seems clear that simple models should not be expected to give useful predictions of vertical velocities, particularly where the flow over irregular topography cannot be specified.

On the limited evidence provided by a few high climbs it seemed that the wave amplitude did not decrease with height as much as predicted by the Scorer model. The vertical velocities predicted by Casswells graphical method showed some agreement with observed values, but this may have been by chance because predictions of unusually strong or weak lift seemed to be contradicted by observations.

Deductions from simple models: One of the values of simple models is that while calculating the resonant wavelength it is possible to get an indication of the stability of any tropospheric wave pattern.

Vergeiner and Lilley (1970) observed that when waves were stationary and of large amplitude the computed and observed wavelengths showed good agreement and were insensitive to small changes in the upstream conditions. When the sensitivity was high the observed patterns were erratic and non stationary.

Starr and Browning (1972) studied the wave pattern to lee of the Welsh hills using high powered radar and found that on some days the wave pattern was stable for several hours while on other days the pattern was complex and unsteady. Although it was necessary to interpolate between published upper air soundings to derive l^2 profiles for the two days quoted it seemed that on the day of stable waves there was a classic l^2 profile, but on the day of unstable waves the layer containing the essential high values of l^2 was not well defined.

Results of the Survey: The survey showed that the simple tropospheric models predicted lee waves on 92% of the days when waves were actually reported. When the profiles showed well defined stable layers of requisite depth pilots reported that soaring conditions were straightforward and waves were observed to extend over a considerable area. If the air was sufficiently moist satellite pictures showed well defined wave trains similar to those described by Cruette (1976). It was common to find a shallow convective layer on wave days; when the depth of convection increased the number of pilots reporting easy climbs seemed to decrease and fewer sites reported waves. On 8% of occasions the simple models failed to predict waves. This was because the l^2 profile was the reverse of that required to meet Scorer's criteria. Upper air soundings showed conditions favourable for deep convection and many surface stations observed showers.

Normal wave days and deep convection wave days: The days when three layer models predicted waves were classified as "normal" wave days, even though some of them appeared near to the limit. The remaining 8% of days were far outside the limits and were classified as "Deep convection" days. The classes differ not only in stability but also in wind speeds, as is shown by table 1.

The very large standard deviations at high levels reflects the presence of jet streams near the area on many wave days. The deep convection days had very little vertical shear below the 500 mbar level, in contrast to normal wave days which usually show a considerable shear within the troposphere.

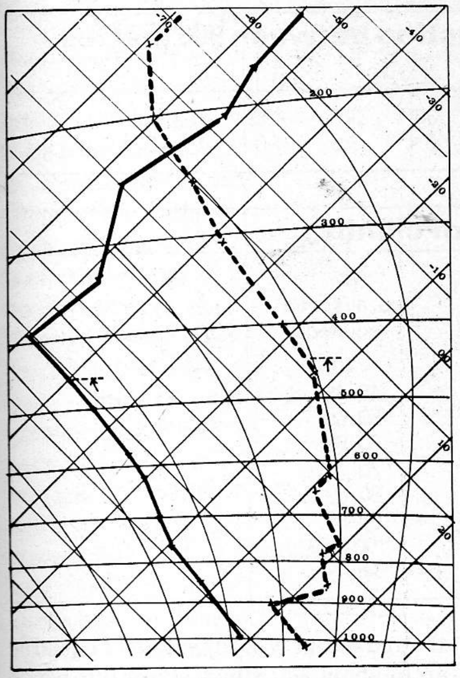
Mean value 871 mb (S.D. 67 mbar)

On the 8% of days classed as "deep convection" the mean value for the base of the stable layer was 469 mb (S.D. 85

Table 1 **Comparison of Wind Speeds on Normal and Deep Convection days**

Speeds in knots with standard deviations in brackets

	900 metres	700 mbar	500 mbar	300 mbar
Normal	27.6 (8.4)	37.1 (11.3)	50.0 (17.0)	69.2 (30.6)
Deep conv.	29.6 (8.5)	30.6 (9.6)	35.3 (13.2)	52.2 (23.3)



3. *Comparison of temperature soundings on normal and deep convection days.*

mbar). On some of these days the increase in the value of l^2 in the stable layer was less than a casual inspection suggested because the increase in winds aloft reduced the effect of increased static stability. Figure 3 shows temperature curves for a normal wave day and a deep convection day. On both occasions pilots reported climbs to nearly 400 mbar.

Although there were a few reports of pilots wave soaring above the tops of passing cumulonimbus clouds most of the climbs which reached the 500 mbar level were well above the tops of nearby clouds. It appeared that the best wave soaring areas were those sheltered from the majority of showers. The largest clouds were observed upwind or far off to the side and most cumulonimbus clouds declined down wind of the major peaks.

The effect of shelter shows up in the tables listing the frequency of high climbs made with different wind directions. The table shows the figures for two sites in Scotland. "A" lies close to the Cairngorm mountains in the central Highlands and "B" lies to the south of this major mountain area. (The areas are marked on fig 1).

Deep convection days occur chiefly with winds from a northerly direction and this table makes it clear that area "A" close to the Cairngorms does not have good wave soaring with northerly winds while area "B" which is sheltered by the mountains to the north experiences many of its best days with northerly winds. It will be appreciated that the alignment of major ridges is also reflected in this table, but area "A" does not lack wave producing ridges for northerly winds.

Differences between wave soaring on "normal" and "deep convection" days: On normal wave days soaring may begin at dawn and a succession of wave climbs may be made throughout the day. Although the position of the best wave lift may vary throughout the day the wave can often be found in the same area for hours on end.

On deep convection days pilots report that the wave is not apparent until cumulus clouds start to form. The best time to start a climb is often limited to a period of two hours or less when the convective cloud is still growing. Once the clouds are well developed it is very difficult for pilots to start a wave climb, but those who are already high can set off on cross country flights with a good prospect of being able to return to base before the day ends, provided they remain high.

Interaction between convection and waves: It was noticed that in mountain areas large masses of convective cloud tend to be maintained in the same area for some time although the individual cloud elements continued to move with the local wind. The persistance of a bank of cloud may be attributable to wave flow set off by the mountains, but when the clouds grow large there is probably interaction between cumulus below and waves above. On days of deep convection the location of waves does not seem to be so closely related with the underlying topography as on normal days. The areas of lift vary erratically and the collapse of a cumulus cloud may coincide with the decrease of wave lift. It is often difficult to pick out the wave pattern, even on satellite pictures, and pilots flying at low levels are often unaware of waves higher up.

Earlier observations of waves above convective clouds: It has long been recognised that waves may exist above cumulus clouds. Diagrams of such flow were given by Gerbier and Beranger (1961) and Bradbury (1963). Jaeckisch (1968) reported waves lying parallel to streets of cumulus cloud and conditions for wave flow over cumulus clouds were described by Kuettner (1972). When waves lay parallel to cloud streets the wind above the cloud top was at right angles to the wind in the convective layer.

Waves at right angles to cloud streets: The existence of waves at right angles to cloud streets was observed by Harrison (1971). Suitable conditions may occur when the convective layer is capped by a well marked stable layer and the wind direction remains constant with height but the speed increases upward. Flight under such a cloud street shows that the distribution of lift is irregular, unlike normal cloud streets where lift can be found along most of the length. Satellite pictures sometimes show the cloud street widens and narrows in phase with the wave pattern above.

Variation of wind direction with height: Lee wave models generally assume that

Table 2 **Base of Stable Layer on Normal Wave days. (Frequency distribution)**

Pressure ranges shown in tens of mbar. Frequencies expressed as percentages.

Below 95	95–90	90–85	85–80	80–75	75–70	70–65	65–60	60–55	Undefined
16.0	13.8	26.8	26.4	11.9	1.8	0.7	0	0.4	2.2

Table 3 **Frequency of Wave Climbs with different wind directions**

	030°	360°	330°	300°	270°	240°	210°	180°	150°	120°
Area «A»	—	—	4.3	21.7	17.4	26.1	17.4	7.2	4.3	1.2
Area «B»	3.9	15.5	25.2	36.1	7.1	6.5	3.9	0.6	1.3	—

the wind direction is sufficiently constant to justify two dimensional treatment of waves. Observations showed that on most days the wind direction did not alter by more than 30° between 900 metres and the top of the climb. A small percentage of days with greater wind shifts were observed but it is likely that the change of wind was the reason for the climb ending at that level. The frequency of various wind changes is shown in table 4.

Table 4 Change of Wind Direction between 900 metres and top of climb

00°	05°	10°	15°	20°	25°	30°	35°	40°	45°	50°	60°
12.0	21.1	16.4	15.4	13.7	8.7	5.4	3.0	1.0	1.7	1.3	0.3

Effect of strong upper winds: The increase of wind with height is one of the features which makes lee waves probable; if the upper winds are very strong a sailplane may be unable to hold station on the forward side of the wave without incurring the penalty of an excessively high sinking speed which may exceed the vertical velocity of the air. The survey shows that high wind speeds were not often a problem except to a few older sailplanes which were outclassed at high speeds. In table 5 column "A" shows the windspeed at the top of the climb and the frequency that these speeds were encountered. Colum "B" shows the indicated airspeed required to hold station flying into wind.

Table 5 Wind speeds at tops of climbs

wind speed (knots)	«A» % of climbs	«B» Ind. airspeed required	% of climbs
<40	34.1	<40	62.8
40–49	20.5	40–49	16.4
50–59	17.4	50–59	9.1
60–69	8.5	60–69	3.8
70–79	8.2	70–79	1.6
80–89	2.5	80–89	–
90–99	0.9	90–99	–
100–109	1.6	100–109	–
N/A	6.3	N/A	6.3

The line marked N/A represents occasions when the exact height was not reported, but the requisite gain of height had been certified. The ratio between indicated airspeed and true airspeed is (p/p_o)

Heights achieved in wave climbs: Table 6 shows the frequency with which different heights were achieved. Only the highest climb on any one occasion has been listed. The table does not distinguish between occasions when the pilot was unable to gain more height and the occasions when the climb was broken off.

A significant number of climbs were halted for lack of oxygen, others were broken off because of increasing cloud cover, lack of daylight, or simply to allow another pilot to fly.

The highest climb of 11 km was made just down wind of mountains which reached 1148 metres. The next highest (9.2 km) had no hills larger than 747 metres nearby.

Reports of unusually light upper winds: Most wave days showed the expected increase in wind speed with height but there were a few reports of pilots encountering much lighter winds than would be expected from the upper air charts and soundings. In some cases the local wind appeared to be calm and pilots could circle in lift without drifting out of the ascending part of the wave.

The existence of strong up currents and apparently calm winds indicates a region where the streamlines are almost vertical. Such areas were usually observed to be very close to the lee slope of a ridge. Steepening of streamlines is associated with waves whose phase line slopes upstream with height. (Smith 1977). Factors which may lead to an increase in wave steepening are a local decrease of wind and also ridges where the lee slope is steeper than the windward slope.

Observations made on flights over Scotland confirmed both these factors were present on occasions of near vertical wave flow. The effect of local steepening of the lee ridge was easiest to observe because the cloud effects associated with the vertical flow only developed along the section where the lee slope was greatest.

Fig. 4 shows a diagram representing the type of flow observed, together with the wind profile estimated form soundings.

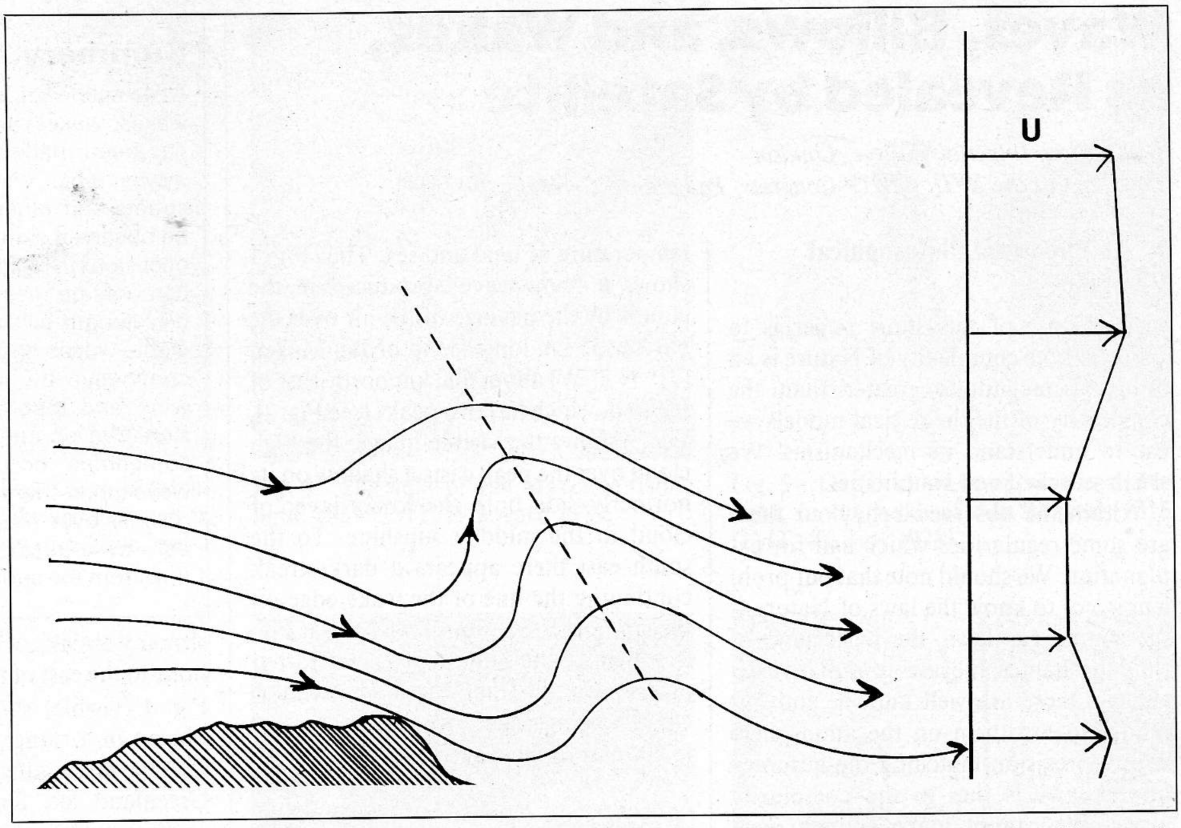
Conclusions: Pilots reports showed that waves have been found over the British Isles in a very wide range of conditions. The average of many soundings showed the air was convectively unstable up to about 870 mbar, above this was a stable layer reaching to approximately 700 mbar. The upper part of the troposphere usually showed a lapse rate near to a saturated adiabat. Wind directions were fairly constant within the wave layer and speeds increased from less than 30 knots at 900 metres to near 70 knots at 300 mbar. The 1^2 profiles were often well represented by a simple three layer model suggested by Wallington; wavelengths calculated from this model were generally in fair agreement with values measured from satellite pictures.

Table 6 Frequency of high wave climbs

(Climbs of less than 3 km excluded)

Height (km)	%
11.0–11.5	0.3
10.5–11.0	–
10.0–10.5	–
9.5–10.0	–
9.0–9.5	0.3
8.5–9.0	–
8.0–8.5	1.3
7.5–8.0	2.5
7.0–7.5	2.5
6.5–7.0	3.8
6.0–6.5	9.1
5.5–6.0	12.6
5.0–5.5	11.7
4.5–5.0	11.7
4.0–4.5	18.0
3.5–4.0	17.4
3.0–3.5	1.6

As the depth of the convective layer increased the numerical models predicted an increase in wavelength and decrease in amplitude until a stage was reached when no waves were expected. Pilots observations showed that although a greater depth of convection increased the difficulty of starting a wave climb it was still possible to climb high, and the vertical velocities were evidently much greater than the models predicted.



4. *Steepening of lee waves with forward slope to phase line and near vertical streamlines. Wind profile on right.*

Tropospheric two or three layer wave models failed to predict lee waves on 8% of days when pilots reported climbs in excess of 3 km. Such occasions were days of deep convection when large cumulus clouds developed and showers occurred.

There seemed to be interaction between convective and wave motions. With shallow convection the development of cumulus was dominated by the wave pattern. As convection became deeper the topographic influence was reduced, waves shifted position irregularly and wave trains became hard to distinguish. The flow patterns which produced cumulus streets lying at right angles to the wave bars above have yet to be explaines.

T.A.M. Bradbury,
Presented at the XVII OSTIV Congress, Paderborn, Germany (1981)

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