# Modelling anabatic flow

*P.C. Manins, CSIRO Australia*
*Presented at the XVII. OSTIV-Congress, Paderborn (BRD), 1981.*

**As a result of direct insolation to a hill slope, adjacent air is able to be warmed quicker than air at the same level over nearby lowlands. The density variation, or buoyancy, of the warmed air produces an anabatic wind up the hillside (see Fig. 1). Such flows are of importance in the ventilation of valleys (Whiteman and McKee, 1978), to understanding the formation of cumulus clouds over mountainous terrain (Orville, 1964), and to the gliding enthusiast (Wallington, 1977).**

*1. Introduction*

Although anabatic winds are unlike katabatic flows in that their influence extends to well above the high land, the physics of each is much the same (Prandtl 1942; see Sutton, 1953). Indeed we can use an approach which represents the anabatic wind as a layer flowing under the influence of its buoyancy relative to a (possibly thermally stably stratified) layer representing the ambient air. This two-layer approach has been used in theories of katabatic flow for some time (e.g. A. Defant 1933, quoted by F. Defant 1949), but a recent theory by Manins and Sawford (1979a) has proved to be more successful than most and its essential features are supported by field measurements (Manins and Sawford 1979b). This theory differs from others primarily by recognising that the growth of the flow with travel distance is due to turbulent entrainment of ambient air. The entrainment has been shown to play a dominant role: it is incorporated in the manner used by Ellison and Turner (1959) for gravitationally stable wall plumes.

Altered by appropriate sign changes the two-layer katabatic flow theory can be applied to anabatic winds. The result is a model to predict the upslope growth of thickness, speed and temperature excess of the anabatic wind.

*2. The Model*

We apply the equations of mass continuity, thermal energy, and the equations of motion as simplified by assumptions of steady two-dimensional two-layer flow to a simple V-shaped hill system (Fig. 2). Although the existence of an ambient thermal stratification will strongly influence the confinement of warmed air to the slope, it can be shown that for hill slopes of length a few kilometres or less under typical conditions, the model for such a simple geometry has an analytic description. It is

$$U = (As)\frac{1}{3} = \left( \frac{\sin\alpha + \frac{5}{16} E \cos\alpha}{\frac{5}{4}\, E + C_D} \cdot \frac{gH}{pCp0} \right)^{\frac{1}{3}} s^{\frac{1}{3}}$$

$$\delta 0 = \frac{H/pC_p}{\frac{3}{4} \cdot E} \cdot (As) - \frac{1}{3}$$

$$h = \frac{3}{4}\, Es$$

(See Manins and Sawford 1979a).

These solutions show the important properties of the anabatic flow. The thickness, h, is expected to increase linearly with distance up the slope. The entrainment coefficient, E, is given by the observed rate of thickness increase. It should be a function of the opposing influences of ambient thermal stratification and slope angle, $\alpha$, measured from the horizontal. It is discussed further below. The temperature excess, $\delta\theta$, of the anabatic air relative to the ambient air at the same height is shown to be proportional to the surface heat flux, H, and to be smaller if the entrainment coefficient is large. $\delta\theta$ is also expected to decrease weakly with increasing distance upslope. Anabatic wind speed U is shown to increase weakly with increased surface heating and distance upslope. It is expected to decrease if surface drag and turbulent entrainment are large. These are represented by coefficients $C_D$ and E.

Although complicated by its influence on the entrainment coefficient, slope angle, $\alpha$, should affect anabatic flow as given by the solutions. Increased steepness leads to faster flow and reduced temperature excess. However, no direct affect on flow thickness is predicted. Clearly, it is essential to understand the behaviour of the entrainment coefficient. For downslope flows E is roughly proportional to tan $\alpha$, approaching zero for horizontal flow and approximately 0.08 for vertical flow. The latter value corresponds to the limit of a free plume under gravity. For gravitationally unstable upslope flows we would expect E to *increase* as slope angle becomes small and

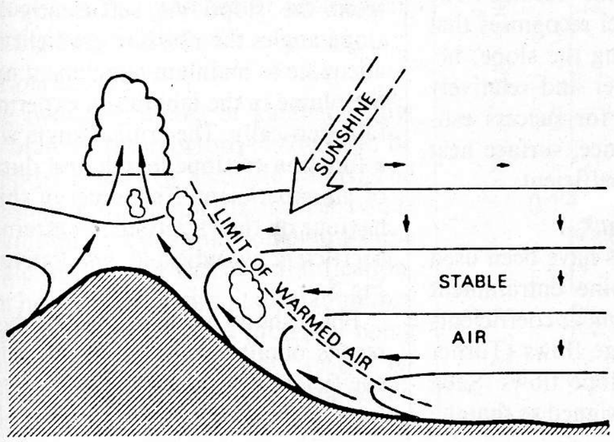


*Figure 1*

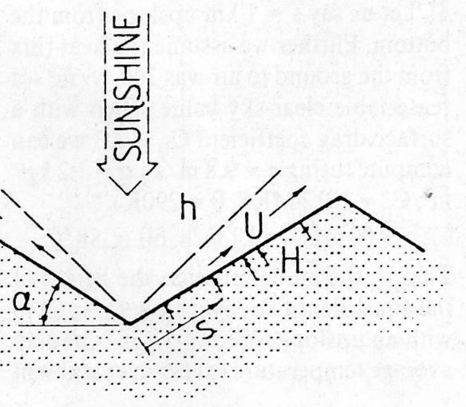


*Figure 2*

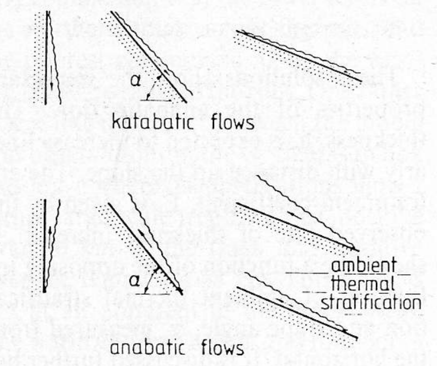


*Figure 3*

to be strongly affected by the confining effects of ambient stratification. (See Fig. 3).

*3. Application*

Let us assume E to be known to be little different from the value for a vertical plume in the application now considered. That is we take $E \simeq 0.08$, assuming the effects of slope angle and ambient stratification cancel each other out. So far as I am aware the only direct observations of what are said to be anabatic winds have been presented by Defant (1949) from unpublished work by A. Riedel in the Innsbrucker Nordkette. An experiment on slopewinds in the same region has recently been undertaken (Freytag und Hennemuth, 1979) but awaits analysis.

Although the data are incomplete and the observed winds by Riedel probably indicate processes additional to local anabatic effects, let us apply the simple model of an anabatic plume. Defant (1949) reports the slope to be $\alpha = 43°$, and since katabatic as well as anabatic winds were observed at different times, presumably the observations were taken about halfway up the Nordkette (see Fig. 4). Let us say s = 1 km upslope from the bottom. Further we assume the heat flux from the ground to air was 200 W/m² - a reasonable clear-sky value. Then with a surface drag coefficient $C_D \simeq .03$ we can compute (using g = 9.8 m/s², $\varrho$ = 1.2 kg/m³, $C_p$ = 1012 MKS, $\theta$ = 290K).

$$h = 60\,m,\ U = 3.1\,m/s,\ \delta\theta = .88\,°C$$

That is, the model predicts the height of the equivalent two-layer flow to be 60 m with an upslope speed of 3.1 m/s and an average temperature excess over ambient

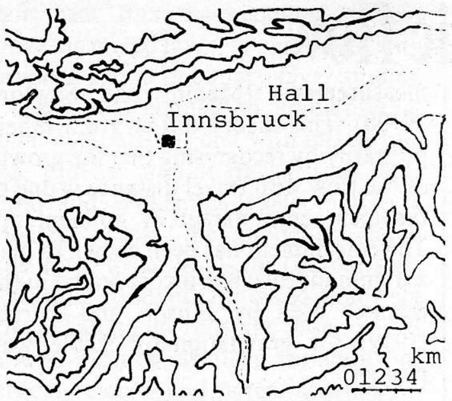


*Figure 4*

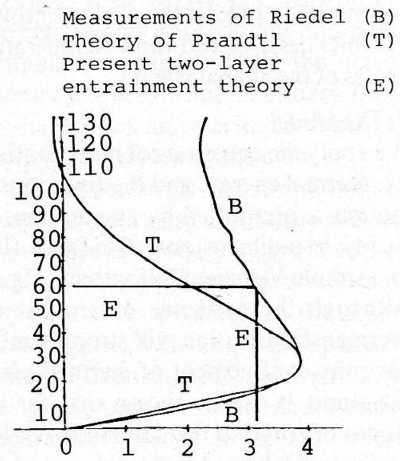


*Figure 5*

of 0.9 °C. Comparison with Fig. 5, from Defant (1949), shows good qualitative agreement for height and speed. There are no data for temperature.

Note that the theory of Prandtl has been fitted by Defant in Fig. 5. For success the local observations of wind speed maximum and height are used. Then an eddy mixing coefficient results. The present two layer model recognises that the wind develops along the slope, becoming thicker, stronger and relatively less warm. It requires for success estimates of upslope distance, surface heat flux and entrainment coefficient.

*4. Laboratory experiments*

Laboratory experiments have been used in the past to determine entrainment rates and hence entrainment coefficients for plumes and drainage flows (Turner 1973), but not for upslope flows. Such experiments must be designed so that the slope can be readily changed and the ambient fluid readily density stratified. This is most easily achievable if water is used as the working fluid, with dissolved salt providing density changes. It is more convenient if the experiments are conducted upside down with heavy salty water from a long narrow slit running *below* a broad slope.

Thus while we do not model the heating of the slope, the approximately two-dimensional (in the mean) plume which issues from the discrete source should grow in a similar way to real anabatic flow. A measure of the angle of spread with distance along the slope is then a measure of the entrainment coefficient. The facility as used in the present experiments is sketched in Fig. 6.

Laboratory experiments done so far have only involved a uniform ambient fluid. Density stratification is to follow at a later date. Fig. 7 shows a typical experimental realization. The large starting vortex at the head of the flow is still visible. Further back towards the source (top left in Fig.7) the flow has become steady, entraining at a uniform rate.

It is interesting to see how the plume, which initially falls vertically, attaches to the slope and approaches steady flow. The plume, being a region of slightly lower pressure than that in the ambient fluid, entrains initially from both sides. The fluid between the plume and the slope is soon all entrained into the plume and so the plume is deflected until it runs along the slope. This occurs quickly near the source but takes longer further along the slope. The phenomenon is called the Coanda effect (Kadosch, 1967).

The lower pressure in the flow opposes gravity which acts to lift the flow from the slope. At sufficiently small slope angles the pressure gradient is not adequate to maintain attachment and so the plume in the laboratory experiments falls vertically. The critical angle will be a function of slope length and duration of an experiment. The effect of this behaviour on the variation on entrainment coefficient is indicated qualitatively in Fig. 8.

Fig. 8 shows a preliminary analysis of results obtained so far for entrainment coefficient as a function of slope angle. Note that data for angles less than 40° to

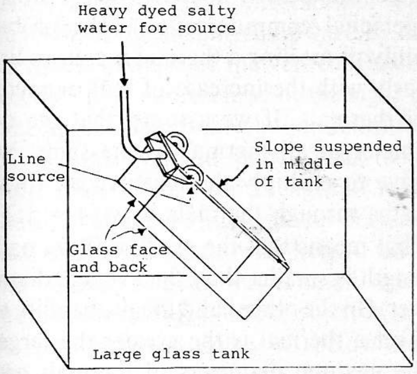


*Figure 6*

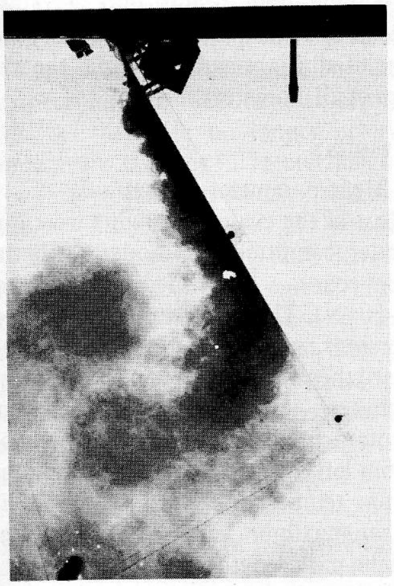

*Figure 7*

the horizontal have not yet been achieved due to the extreme sensitivity of the set-up for the reasons mentioned above. Of course the values for entrainment coefficient shown in Fig. 8 will be reduced in the presence of an ambient density stratification.

### *5. Conclusion*

The two-layer model of katabatic flow by Manins and Sawford (1979a) can be adapted readily to anabatic winds. The missing information, viz the behaviour of entrainment as a function of hill-slope angle and abient density stratification, can be obtained from suitable laboratory experiments. These are presently being undertaken and preliminary results reported here show encouraging possibilities.


### References

*Defant, F.* 1949, Zur Theorie der Hangwinde, nebst Bemerkungen zur Theorie der Berg- und Talwinde. Arch. Met. Geoph. Biokl., Al, 421-450.

*Ellison, T.H.* and *Turner, J.S.* 1959 Turbulent entrainment in stratified flows. J. Fluid Mech., 6, 423-448.

*Freytang, C.* and *Hennemuth, B.* (eds.) 1979 Hangwind experiment Innsbruck Oktober 1978 - Datensammlung. Münchener Universitäts Schriften, Fakultät für Physik, Wissenschaftliche No. 36.

*Kadosch, M.* 1967 The curved wall effect. Second Cranfield Fluidics Conference, 3-5 Jan 1967, Cambridge, U.K. Paper A4.

*Manins, P.C.* and *Sawford, B.L.* 1979a A model of katabatic winds. J. Atmos. Sci., 36, 619-630.

*Manins, P.C.* and *Sawford, B.L.* 1979b Katabatic winds: A field case study. Quart. J.R. Met. Soc., 105, 1011-1025.

*Orville, H.D.* 1964 On mountain upslope winds. J. Atmos. Sci., 21, 622-633.

*Sutton, O.G.* 1953 Micrometeorology. McGraw-Hill, London.

*Turner, J.S.* 1973 Buoyancy Effects in Fluids. C.U.P. Cambridge.

*Wallington, C.E.* 1977 Meteorology for Glider Pilots Third Edn. John Murray, London.

*Whiteman, C.D.* and *McKee, T.B.* 1978 Air pollution implications of inversion descent in mountainous valleys. Atmos. Environment, 12, 2151-2158.


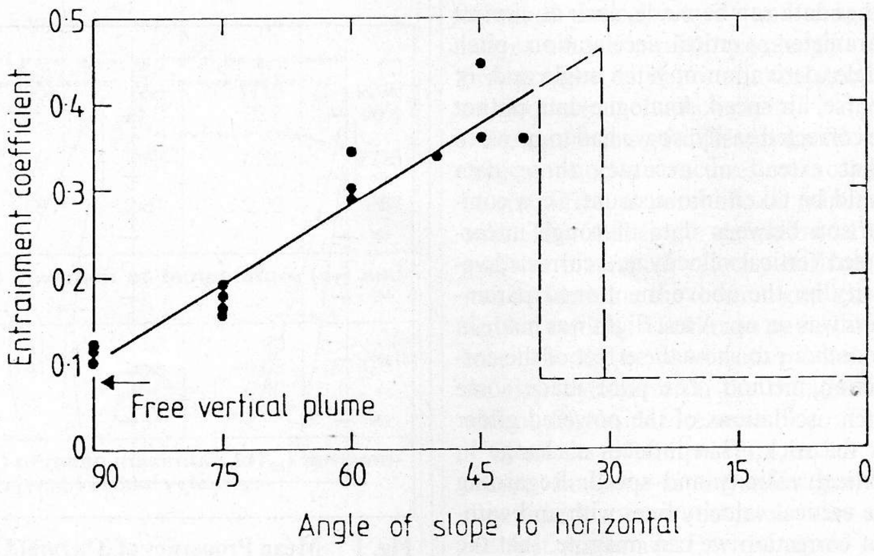


*Figure 8*


## Zusammenfassung:

*Modell einer anabatischen Strömung*

Die Luft an Berghängen erwärmt sich – bedingt durch die direkte Sonneneinstrahlung – schneller als die Luft in derselben Höhe über dem Flachland. Die Dichtevariation oder der Auftrieb der erwärmten Luft verursacht einen anabatischen Wind den Hang hinauf. Solche Strömungen sind für die Ventilation von Tälern, für das Verständnis der Bildung von Cumuluswolken über Gebirgsregionen und für den Segelflugenthusiasten wichtig.

Obwohl die anabatischen Winde in ihrer Höhenausdehnung stärker wirksam sind als die katabatischen Winde, ist die Physik von beiden Vorgängen fast gleich. Deshalb kann man eine Näherung benützen, welche den anabatischen Wind als eine Strömungsschicht unter dem Einfluss des Auftriebs relativ zur Schicht der umgebenden Luft ansieht. Zunächst wurde diese Zwei-Schichten-Näherung in der Theorie der katabatischen Strömung benutzt, aber eine neue Theorie von Manins und Sawford (1979b) versprach mehr Erfolg und ihre Hauptcharakteristiken werden durch Feldmessungen erhärtet.



*P.C. Manins, CSIRO Australia.*


*Presented at the XVII OSTIV Congress, Paderborn, Germany (1981)*