The Effects of Thermals and Wind on the Distribution of Atmospheric Pollution

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Zusammenfassung:

Die Auswirkungen von Thermikentwicklung und Windfeld auf die Verteilung atmosphärischer Spurenstoffe

In einer Studie über die Auswirkungen von Thermikentwicklung und Windfeld auf die Verteilung atmosphärischer Verunreinigungen wird ein Box-Modell benützt, in dem die Spurenstoffe mit einer konstanten Emissionsrate pro Flächeneinheit an der Oberfläche angesetzt werden.

Die Box wird so flach angesetzt, dass sofort eine homogene Verteilung der emittierten Spurenstoffe in der Höhe zwischen Boden und Deckel angenommen werden kann.

Folgende Effekte werden untersucht:

1) Verringerung der Konzentration der Spurenstoffe durch Thermikeinwirkung
2) Horizontale Durchströmung der Box
3) Leeseitige Auswirkungen der emittierenden Oberfläche

Die Werte von Emission, Quellengrösse, thermischer Auswirkung und Windfeld für die Stadt Buenos Aires werden herangezogen, um die Grössenordnung der Effekte zu studieren.

1. Introduction

Box models of the transport and dispersion of atmospheric pollution are very simple and coarse, but they are nonetheless useful to study the nature and importance of the major factors controlling the mean concentration of pollution.

Fig. 1 illustrates the simple box model we use in this study. It is assumed that contaminants are emitted at a steady rate, *E*, from a rectangular ground surface, as shown in the figure. The depth, *h*, of the box is considered to be shallow enough for the contaminants to be instantly distributed uniformly with height. Two simple processes of removing contaminants from the box are considered, namely:

(1) vertical extraction of contaminants by thermals rising, at speed *T* up through a proportion *A* of the area of the top of the box, and

(2) a horizontal wind, *W*, bringing clean air into the box and carrying contaminants out.

The effect of the length, *L*, of the box, in the alongwind direction of the contaminant source is also studied.

2. Characteristic Situations

Let us consider four types of situations:

2.1 *No horizontal wind or thermals* $(W = T = 0)$

Assuming the emission starts, at a time t_0, and continues at a constant rate the increase in mass, *M*, of contaminants within the box must increase steadily with time, and the mean concentration within the box must also increase steadily—concentration, *C*, being the mass, *M*, divided by the fixed volume, *V*. More formally,

$$\frac{dC}{dt} = \frac{d}{dt}\left[\frac{M}{V}\right] = \frac{1}{V}\left[\frac{dM}{dt}\right] = \frac{E \text{ x Area of emission}}{\text{Volume of the box}} = \frac{E}{h} \qquad (1)$$

where *E* = the rate of emission of contaminant per unit time per unit area of the ground surface of the box.

$$\text{Thus} \quad C_t = C_o + \frac{E}{h}t \qquad (2)$$

where C_0 and C_t = the mean concentrations in the box at times t_0 and *t*.

The steady increase of concentration with time is shown in Fig. 2.

2.2 *Thermals, but no horizontal wind*

Consider an oversimplified, but nonetheless useful postulate that some of the contaminants are carried out of the box in thermal up-curents while clean air is brought down into the box by compensation downdraughts. Let us summarise the net effect by taking the thermal outflow as a uniform vertical current of speed *T* over a proportion *A* of the surface area of the top of the box, as shown in Fig. 1.

Taking the box to be of unit acrosswind width, we can write

$$\frac{dC}{dt} = \frac{1}{V}\left[\frac{dM}{dt}\right] = \frac{1}{V}\left[EL - TALC\right] = \frac{E}{h} - \frac{TAC}{h} \qquad (3)$$

$$\text{Thus} \quad C_t = C_o \exp(-Qt) + \frac{E}{Qh}\left[1 - \exp(-Qt)\right] \qquad (4)$$

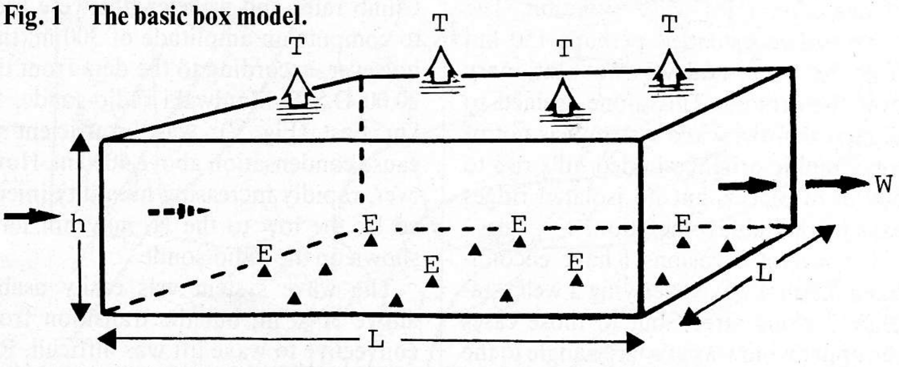
where $Q = \dfrac{TA}{h}$ and C denotes the mean concentration within the whole box.

The second term expresses the tendency towards the rate of emission being balanced by the rate of extraction by thermals; in the limit, as $t \to \infty$, the mean concentration of contaminants within the box tends to E/Qh, or E/TA. Fig. 2 shows the exponential approach to this limit.

2.3 *A horizontal wind, W, but no thermals*

As the clean air coming into the box (of unit crosswind width) at speed *W* sweeps over a narrow crosswind slice δx of the source area in a time increment δt, the

Fig. 1 The basic box model.



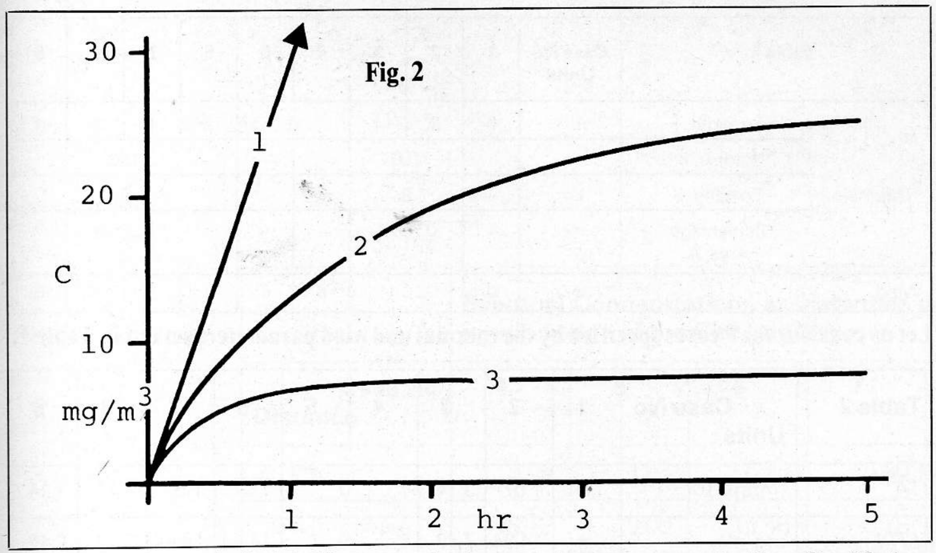
E denotes the emission of contaminants at a steady rate uniformly over the ground surface area of the box.

h and L are the respective depth and alongwind length of the box

W denotes the wind through the box

T denotes thermal updraughts from a proportion, *A*, of the upper surface area of the box.

Fig. 2



The straight line, labelled 1, illustrates the increase in mean concentration, C, with time when there is no wind and no thermals. Curves 2 and 3 are concentration versus time curves for thermals of weak and moderate strength. Values on the axes and numbers on the curves are for the Buenos Aires examples 1, 2 and 3 detailed in Section 4 of this paper.

rate of change of concentration and the rate of change of volume into which the contaminant is distributed will be related by the equation

$$\frac{\partial M}{\partial t} + W\frac{\partial M}{\partial x} = \frac{\partial}{\partial t}\left[Ch\delta x\right] + W\frac{\partial}{\partial x}\left[Ch\delta x\right] = E\delta x \qquad (5)$$

$$\frac{\partial C}{\partial t} + W\frac{\partial C}{\partial x} = \frac{E}{h} \qquad (6)$$

and when a steady state $\frac{\partial C}{\partial t}$ = 0) is attained

$$C_x = C_o + \frac{Ex}{Wh} \qquad (7)$$

where C_0 and C_x are now the concentrations at distances of 0 and x downwind from the upwind edge of the course.

At the downwind end of the source no more contaminant is emitted into the box from the underlying surface. So, from the downwind edge of the source onwards the concentration remains constant, equal to *EL/Wh.*

Fig. 3 shows the distribution with distance along wind.

2.4 *A horizontal wind, W, and thermals*

Now, when a steady state has been attained, we have the rate of change of concentration across a thin vertical slice of the box given by

$$W\frac{\partial}{\partial x}\,Ch\delta x \;=\; E\delta x - TAC\delta x \qquad (8)$$

$$\therefore\;\frac{\partial C}{\partial x} = \frac{E}{Wh} - \frac{TAC}{Wh} \qquad (9)$$

for which the solution is:

$$C_x = (C_o + \frac{E}{TA})\left[1 - \exp(-\frac{TA}{Wh}x)\right] \qquad (10)$$

of, if $C_o = 0$,

$$C_x = (E/TA)\left[1 - \exp - TAx/Wh)\right] \qquad (11)$$

The asymptotic concentration as $x \to \infty$ is *E/TA*, i.e. the extraction by thermals eventually balances the emission rate. At the downwind edge of the box

$$C_L = (E/TA)\left[1 - \exp(- TAL/Wh)\right] \qquad (12)$$

Downwind from the downwind edge of the box

$$C_x = C_L\exp(-\frac{TA}{Wh}x) \qquad x \geq L \qquad (13)$$

Fig. 3 shows the distribution of C_x with distance along and downwind of the box.

3. Principal Factors

Although the box models are only rough approximations to real situations, they are adequate to highlight the principal factors determining maximum or eventual concentrations.

With no wind or thermal convection, the principal factors are *E/h* and, of course, time. The higher the value the more intense the concentration becomes with time, and there is no ventilation or extraction flow to limit the increase in concentration.

With thermals the increase in concentration is limited to a maximum of *E/TA* when the emission is balanced by the vertical extraction.

With a steady horizontal wind but no thermals, the concentration across and downwind of a source settles down to a steady state with its value increasing linearly across the source and remaining constant from the downwind edge onwards. The principal factor determining the maximum added concentration is *EL/Wh,* which gives the concentration at the downwind edge of the box.

With both thermals and a wind, the rate of increase of concentration across a source is less than that for the wind only situation, because the thermals are at work extracting contaminants through the top of the box. From the downwind edge of the source the steady state concentration decreases exponentially. The principal factors in the situation are *E/TA* representing the opposing effects

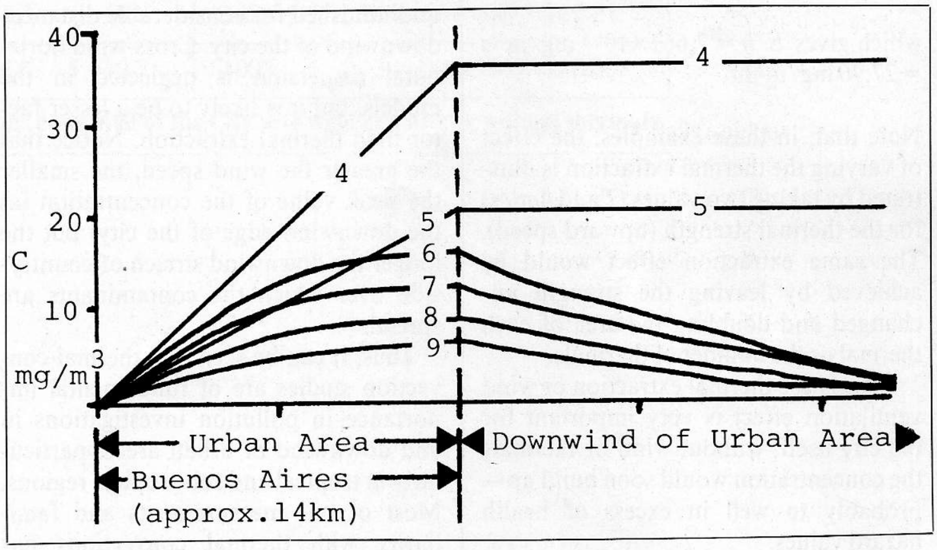


Fig. 3 shows the distribution with distance along wind.

of emission rate and thermal extraction, and TA/Wh indicating the opposing effects of thermal extraction and ventilation by wind. Notice that, downwind of the box, the wind ventilation is now an unfavourable factor in the decrease in concentration with distance; the stronger the wind and higher the top of the box the slower the rate of decrease of C with x. On the other hand the higher the value of Wh, the lower will be the maximum value, C_L, given by Equation (12).

4. Application of the Models to a City

The city of Buenos Aires covers an area of about 200 square kilometers. It uses an average of 2,099,282 cubic metres of fuel per year. Taking the mean density of the fuel as 0.7 tons per cubic metre we get

Annual emission for the city 1,469,498 tons

Average emission per square metre per second,

$E = 0.233$ milligrams.

As the city is approximately square, we take the downwind length as

200 km, i.e.
$L = 14.142$ km

Taking the tenth floor level of a building as the approximate level to which the contaminants are well mixed, let us set

$h = 30$ metres

which gives $E/h = 7{,}667 \times 10^{-3}\ \mathrm{mg/m^3\ s} = 27{,}96\ \mathrm{mg/m^3\ hr}$

Note that, in these examples, the effect of varying the thermal extraction is illustrated by taking two values (2 and 4 m/s) for the thermal strength (upward speed). The same extraction effect would be achieved by leaving the strength unchanged and doubling the area of each thermal or the number of thermals.

Clearly the thermal extraction or wind ventilation effect is very important for the city itself; without wind or thermals the concentration would soon build up—probably to well in excess of health hazard values.

The values shown for distances across and downwind of the city are the eventual values that would be reached if a steady state were to be attained. They do not show downwind variations, but they nonetheless include a feature that is sometimes neglected in urban pollution studies; namely the far reaching drift of contaminants downwind of urban areas. It is apparent that, without thermals, the urban pollution levels can spread almost undiminished for considerable distances downwind of the city. Cross-wind horizontal dispersion is neglected in the models, but it is likely to be a lesser factor than thermal extraction. Notice that the greater the wind speed, the smaller the peak value of the concentration (at the downwind edge of the city) but the longer the downwind stretch of countryside over which the contaminants are spread.

Thus, it can be seen that thermal convection studies are of fundamental importance in pollution investigations in and downwind of urban areas, particularly in tropical and sub-tropical regions. Most of our measurements and familiarity with thermal convections has come from measurements and observations using gliders or motorgliders over open countryside. More of such measurements and observations are needed over large urban areas.

Table 1		**Case No. Units**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Thermals	Strength, T	m/s	0	2	4	0		2	4	2	4
	Radius, R	m	—	150		—	—	150			
	Spacing, S	km	—	3		—	—	3			
	Percentage Area, A	%	—	0,785%		—	—	0,785%			
Wind		m/s	0	0		3	6	3	3	6	6

Let us consider the 9 cases specified by the thermal and wind parameters set out in Table 1.

Table 2	**Case No Units**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
TA	$(m/s)\times10^{-2}$	0	1.57	3.14	0		1.57	1.57	3.14	3.14
E/TA	mg/m^3	∞	14.84	7.42	∞		14.84	14.84	7.42	7.42
TA/h	$(1/s)\times10^{-4}$	0	5.233	10.47	0		5.233	5.233	10.47	10.47
Wh	m^2/s	∞			90	180	90		180	
E/Wh	$(mg/m^4)\times10^{-3}$	∞			2.589	1.294	2.589		1.942	
TA/Wh	$(1/m)\times10^{-4}$	–	∞		0		1.749	0.872	3.489	1.749

Table 2 lists the values of particular factors in the equations.

Concentrations at several times from the start of emission in a no wind and no thermal situation and for distances across and downwind of the city for thermal and or wind are listed in Table 3. All of these examples take the initial concentration , C_0, to be zero.

	Mean Concentrations in the City (mg/m³)		
Case No Time	**1**	**2**	**3**
After 1 hr	27.96	12.59	4.53
After 2 hrs	55.92	14.50	6.29
After 3 hrs	139.80	14.83	7.24
After 10 hrs	279.6	14.84	7.25
After 20 hrs	559.2	14.84	7.25

Table 3(a): Concentrations in the City with no wind and with or without thermals.

	Eventual Concentrations at Distances downwind from the upwind edge of the City (mg/m^3)					
Case No **Distance**	**4**	**5**	**6**	**7**	**8**	**9**
3.5 km	9.06	4.53	6.78	3.90	5.23	3.90
7 km (midway)	18.12	9.06	10.46	6.78	6.77	5.23
10.5 km	27.18	13.59	12.46	8.90	7.23	6.23
14.14 km (downwind edge)	36.6	18.3	13.58	10.51	7.37	6.79

Table 3(b): Concentrations in the City with wind and with or without thermals.

	Eventual Concentrations and Distances downwind from the downwind edge of the City (mg/m^3)					
Case No **Distance**	**4**	**5**	**6**	**7**	**8**	**9**
1 km	36.6	18.3	11.41	9.63	5.20	5.70
2 km	36.6	18.3	9.58	8.83	3.67	4.79
5 km	36.6	18.3	5.68	6.80	1.29	2.84
10 km	36.6	18.3	2.37	4.39	0.23	1.19
20 km	36.6	18.3	0.42	1.84	0.007	0.21
50 km	36.6	18.3	0.002	0.13	2×10^{-7}	0.001

Table 3(c): Concentrations downwind of the City with wind and with or without thermals.